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THE MOST FAMOUS EQUATION*

A typical undergraduate physics textbook, written by Joseph W. Kane and Morton M. Sternheim,[1] says:

> The equation $E = mc^2$ is perhaps the most famous equation of twentieth-century physics. It is a statement that mass and energy are two forms of the same thing, and that one can be converted into the other (*ibid.*, p. 493).

With the first of these sentences, concerning the fame of Albert Einstein's equation, it is difficult to disagree. But the second sentence, which characterizes mass and energy as interconvertible and "two forms of the same thing," suggests that some care will need to be exercised in interpreting the famous equation. Indeed, it is difficult to find a scientific equation whose ontological implications have been misunderstood so widely and in so many ways. I shall begin by looking at some of the issues that arise when interpreting the famous equation. Along the way, I shall examine briefly some of the ways in which one could become confused by the equation's common interpretations—not only in ordinary physics texts, but also by some very distinguished physicists and philosophers. I shall then try to set the record straight.[2]

I. SOME ISSUES

Let us begin to lay out some of these interpretive issues by taking seriously the textbook's remark that mass "can be converted into"

* My thanks to Arthur Fine for comments on the penultimate draft of this essay, and to Alexander Rosenberg for his encouragement.

[1] *Physics* (New York: Wiley, 1978).

[2] To keep things simple and focused, I shall restrict myself throughout to interpretation of the special theory of relativity, leaving aside considerations arising from the general theory, quantum mechanics, and so forth.

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energy and vice versa. Strictly speaking, this does not mean merely that when mass m disappears, a quantity E of energy takes its place, where $E = mc^2$. To say that some mass is "converted" into energy is to say that the mass *turns into* the energy that replaces it—that there is then a certain parcel of energy that used to be the mass.

For example, after a radioactive nucleus decays, there is often said to be a "mass defect": the sum of the masses of the daughter bodies is less (by Δm) than the mass of the original nucleus. Some (Δm) of the original mass is said to have been "converted" into the kinetic energy E of the daughter bodies, where $E = (\Delta m) c^2$. Since c (light's speed in a vacuum) is so large, a very small mass can be "turned into" a great deal of energy. For instance, when a tritium nucleus (one proton, two neutrons) decays into a helium-3 nucleus (two protons, one neutron) along with an electron and an antineutrino, the tritium's mass exceeds the sum of the products' masses by a small quantity that is "equivalent" to about 0.0186 million electron volts of energy. Has mass turned into energy, or merely disappeared and been replaced by an "equivalent" quantity of energy?

In classical physics, at least, a body's mass is often characterized as its "quantity of matter."[3] Mass, on this view, is a property, whereas matter is a stuff, a substance. If mass is the quantity of matter, then perhaps the physics text should have said that *matter* is converted into energy. Among the physical processes called to mind by this interpretation are a caterpillar being transformed into a butterfly and liquid water turning into steam.

Energy, on this interpretation of the famous equation, would have to be like matter in being a stuff. Classical physics sometimes seems to treat energy in that way. Occasionally, energy conservation is stated as the principle that energy is "neither created nor destroyed" rather than as the principle that an isolated system's total energy remains constant. A parcel of energy is sometimes described as being converted from one form into another or as moving from one place to another—for example: as originally being stored as chemical energy in a battery, then passing through the electric and magnetic fields, eventually passing to a resistor (perhaps a flywheel, which possesses the parcel as kinetic energy), and finally (let us suppose) passing via

[3] "*Quantity of matter is a measure of matter that arises from its density and volume jointly*.... I mean this quantity whenever I use the term 'body' or 'mass' in the following pages"—Definition 1 in Isaac Newton, *The* Principia: *Mathematical Principles of Natural Philosophy*, I.B. Cohen and A. Whitman, trans. (Berkeley: California UP, 1999), pp. 403-04.

friction to the surrounding air molecules as heat energy (that is, as the air molecules' kinetic energies of random motion).

This view of energy gained support in the late nineteenth century. Like other forms of potential energy, electric or magnetic potential energy had originally been considered as having an arbitrary zero level and as being possessed by a system as a whole rather than as having some definite distribution among the system's constituents. But with the rise of James Clerk Maxwell's electromagnetic theory, electric and magnetic potential energy were reinterpreted as electromagnetic field energy. Accordingly, a field of zero strength was taken to possess zero energy, and Maxwell's electromagnetic theory determined a field's energy density in space. All of this supported the view of energy as a substance that can flow; such motion requires that the energy at one place and time be the self-same thing as the energy at a certain other place and time.

On the other hand, many notable followers of Maxwell (such as Heinrich Hertz and Oliver Heaviside[4]) had grave doubts regarding what Heaviside called the "thingness" of energy. To mention just one source of disquiet: Maxwell's equations fail to fix uniquely the vector describing the flow of energy (specifically, the energy-flux density) through the electromagnetic field. The flux of the Poynting vector over an entire surface enclosing a volume is ordinarily interpreted as giving the rate at which electromagnetic-field energy is leaving the enclosed volume. But numerically the same result is given by the flux of the Poynting vector plus any vector field possessing zero divergence. While these two formulas for the energy-flux density agree on the net energy flow into or out of a given volume, such as the volume enclosed by a given spherical surface, the two formulas may disagree regarding the energy flow across a given open surface, such as the spherical surface's top hemisphere. (For instance, if the divergence-free vector field is characterized at every point by the same upward-pointing vector, then it portrays some additional energy as flowing into the volume through the bottom surface, as well as an equal additional energy flow out of the volume across its top surface. These two flows together have no net effect on the total enclosed energy.) The energy flow across an open surface is left entirely undetermined by classical electromagnetic theory; it might be termed "superfluous theoretical structure." Perhaps, then, there is no fact of the matter regarding the energy flow across an open surface—because energy is

[4] *Electrical Papers, Volume 2* (Boston: Copley, 1925), p. 527.

not a stuff (any more than velocity is).[5] Likewise, perhaps a body's mass is not the quantity of its matter, but rather merely a quantitative property determining the body's resistance to being accelerated by a force. (This was Maxwell's[6] own understanding of mass.) On this interpretation of mass and energy, we could not join Elie Zahar[7] in interpreting Einstein's famous equation as characterizing the underlying identity of what ordinarily appear to be two distinct substances:

> According to the accepted mechanistic view, ordinary "ponderable" matter, i.e., the extended hard "stuff" with which we are all familiar, constitutes the most fundamental layer of physical reality.... [M]atter and energy were both conserved, but each separately from the other. Einstein showed that the two levels could be regarded as identical.... The stuff which appears to the senses as hard extended substance and the quantity of energy which characterises a process are in fact one and the same thing (*ibid.*, pp. 261-62).

If energy and mass do not indicate quantities of stuffs the parcels of which have continuing identities over time, stuffs that can be transformed into one another in something akin to the sense in which a caterpillar turns into a butterfly, then it is difficult to see how there could be any reasonably strict sense in which mass *turns into* energy.

Let us turn to another interpretive issue. Our physics text said that $E = mc^2$ "is a statement that" mass can be converted into energy and vice versa. But whether there happen to be, or even could physically possibly be, any processes that make mass disappear, putting energy in its place, is a different question from what the "rate of exchange" between mass and energy would be in such processes. For instance, that certain nuclei are unstable, and that some mass is replaced by an "equivalent" quantity of energy when those nuclei decay, is a result of other laws, not just $E = mc^2$.

That $E = mc^2$ tells us nothing about whether any physical processes allow energy and mass to be interconverted is exploited by Margaret Morrison[8] in a recent passage interpreting our favorite equation:

> Does [$E = mc^2$] mean that mass and energy are the same thing?... [I]n order for this relation to imply a complete identity, it seems that a total

[5] I examine this issue and a host of related problems in my *Locality, Fields, Energy, and Mass: An Introduction to the Philosophy of Physics* (Malden, MA: Blackwell, forthcoming), chapter 5.

[6] *The Scientific Letters and Papers of James Clerk Maxwell, Volume 2,* P.M. Harman, ed. (New York: Cambridge, 1995), pp. 396, 811; and Harman, "Edinburgh Philosophy and Cambridge Physics," in Harman, ed., *Wranglers and Physicists* (Manchester: University Press, 1985), pp. 204-24, especially pp. 222-23.

[7] *Einstein's Revolution: A Study in Heuristic* (La Salle, IL: Open Court, 1989).

[8] *Unifying Scientific Theories* (New York: Cambridge, 2000).

> conversion of mass into energy should be possible. However, such a conversion is not possible, because of the conservation of baryon number, which says that the total number of these particles cannot change; they can be converted into other baryons, but cannot disappear entirely. Baryons include, among other things, protons and neutrons, which make up most of the mass of ordinary matter, and therefore most of the mass of ordinary matter is not available for conversion into energy.... So we are left in the rather odd position of asserting an equivalence between mass and energy, but not an identity that would allow one to be fully transformed into the other (*ibid.*, p. 182).

These remarks correctly recognize that $E = mc^2$ is logically consistent with limitations imposed by other laws on the processes that allow mass to be converted into energy. But these remarks may mislead us regarding what these limitations actually are. Baryon number conservation does not prevent baryons (such as protons and neutrons) from being "fully transformed" into energy (or vice versa). A proton has baryon number $+1$ and an antiproton has baryon number -1, so their mutual annihilation (with the creation of photons, quanta of light possessing energy and having baryon number 0) does not violate baryon number conservation. Of course, antiprotons are not "ordinary matter," and so Morrison is correct insofar as she is suggesting that without the assistance of extraordinary matter, most of the mass of ordinary matter is not available for conversion into energy.

Nevertheless, I do not see why Morrison insists that for mass and energy to be the same thing, a total conversion of mass into energy must be physically possible. Even if mass and energy were essentially the same (like water and steam), it could be a natural law that 90% is the maximum fraction of the initial mass that a process can turn into energy. Or it could simply be that there is no process, permitted by the laws of nature, that turns mass entirely into energy, just as it could be that there is no process, permitted by the laws of nature, that enables diamond to be turned into graphite (though both are forms of carbon).

We have now arrived at the central claim of the textbook passage: that according to $E = mc^2$, "mass and energy are two forms of the same thing." Some authors try to slide from the interconvertibility of mass and energy to their identity. Here, for instance, is Max Jammer[9]:

> As we have seen, the conversion of an electron-positron pair, for example, into gamma radiation or its mirror phenomenon is an incontestable experimental confirmation of the assertion of the theory of relativity that

[9] *Concepts of Mass* (Cambridge: Harvard, 1961).

> mass and energy are mutually and completely interconvertible. This state of affairs raises the following questions: Are not the two entities which are interchangeable essentially the same? Is not what is generally spoken of as an equivalence in reality an identity? Are therefore not "mass" and "energy" merely synonyms for the same physical reality, which...may perhaps be termed "massergy" (*ibid.*, p. 184)?

Jammer clearly intends the answer to be "yes," but from the fact that one thing can be converted into the other, it simply does not follow that the two are actually different forms of the same thing. James Prescott Joule, for instance, famously suggested that all forms of energy (heat, kinetic energy, gravitational potential energy, chemical bond energy, electrical energy, and so on) are interconvertible, but he did not go so far as to hold that they are all different forms of the same thing. In 1843, he showed experimentally that (what he called) the "mechanical equivalent of heat" is (in modern units) 4.18 Joules of energy per calorie of heat. In other words, Joule demonstrated that a 1 kg mass falling through a distance of 42.67 cm contributes exactly enough mechanical energy (in doing work by, say, turning a stirring paddle immersed in water) to add 1 calorie of heat to the water. But he saw this "equivalence" as involving how much of one thing replaces a given quantity of the other—their fixed "rate of exchange." He did not understand the mechanical equivalent of heat as we do today: as a conversion factor, that is to say, a way to change our units for measuring the same thing, because heat is just a particular form of energy.[10] Yet this is how "mass-energy equivalence" is often understood. In the words of a fine relativity text by Edwin Taylor and John Archibald Wheeler[11]:

> [J]oules and kilograms are two units—different only because of historical accident—for one and the same kind of quantity, mass-energy.... The conversion factor c^2, like the factor of conversion from...miles to feet, can today be counted, if one wishes, as a detail of convention, rather than as a deep new principle (*ibid.*, p. 137).

As Einstein[12] himself writes:

[10] See Yuhuda Elkana, *The Discovery of the Conservation of Energy* (Cambridge: Harvard, 1974). Einstein writes: "[E]ach absorption or release of energy brings about, respectively, an increase or decrease of the mass of the body involved. Energy and mass appear as equivalent quantities in the same way that heat and mechanical energy do"—"On the Development of Our Views Concerning the Nature and Constitution of Radiation," in *The Collected Papers of Albert Einstein, Volume 2*, A. Beck, trans., pp. 379-94, on p. 386.

[11] *Spacetime Physics* (San Francisco: Freeman, 1966).

[12] *The Meaning of Relativity* (Princeton: University Press, 1953).

> Mass and energy are therefore essentially alike; they are only different expressions for the same thing (*ibid.*, p. 45).

But how is mass-energy equivalence in this ontological sense (not merely in terms of a "rate of exchange") to be understood?

One problem arises from the fact that under a standard interpretation, relativity theory denies the objective reality of various properties that we ordinarily assign to material bodies (such as their length and velocity) and to events (such as their separation in space and their separation in time). Each of these quantities is *frame dependent*; none is "Lorentz invariant"—that is, the same in every inertial frame of reference. Only what is the same in every inertial frame is a genuine feature of reality. The value that any frame-dependent quantity assumes in a given inertial frame reflects not just reality, but also that reference frame's own particular perspective. The Lorentz invariant quantities are exactly those which depend only on how the universe really is, uncontaminated by any contribution from us in describing the universe.

Consider a body's mass m. (Here I mean what is sometimes called the body's "rest mass"—though, as I shall explain, I prefer to call it simply the body's "mass.") This quantity is Lorentz invariant, and hence objectively real. But the body's energy is not, since its energy depends on its speed, and its speed v is plainly frame dependent.[13] In particular, relativity says that a body's energy E is $m\gamma c^2$, where $\gamma = 1/\surd[1 - (v^2/c^2)]$. Indeed, Heaviside made a similar point in a classical context, where a body's kinetic energy is $(\frac{1}{2})mv^2$. One reason that Heaviside doubted energy's substantiality is that a body's energy depends on its velocity and so is frame dependent, and Heaviside believed in what he called "the relativity of motion": that there are in reality no absolute velocities since only what is the same in every inertial frame can be a genuine feature of reality (*op. cit.*, pp. 525-26; 521).[14] Likewise, the path taken by energy flowing in a system, being frame dependent, lacks objective reality.

A body's energy E and momentum p (a vector—I suppress any vector notation—the magnitude of which, in relativity theory, is $m\gamma v$), *taken together*, form an invariant, mc:

$$\surd[(E/c)^2 - p^2] = \surd[(m\gamma c)^2 - (m\gamma v)^2] = m\gamma\surd[c^2 - v^2]$$

$$= m(1/\surd[1 - (v^2/c^2)])c\surd[1 - (v^2/c^2)] = mc$$

[13] At this point, you might wonder how E can possibly equal mc^2 when E is frame dependent but m and c are the same in every inertial frame. Stay tuned.

[14] See also Heaviside, *Electromagnetic Theory, Volume 1* (London: Benn, 1922), p. 75.

To appreciate this invariant, let us use space-time geometry. At a given moment, a body is associated with an energy-momentum 4-vector P. The lengths of P's three spatial components (along the x, y, and z axes) are given by the components p_x, p_y, and p_z of the body's momentum, which equal $m\gamma v_x$, $m\gamma v_y$, and $m\gamma v_z$ (where v_x, v_y, and v_z are, respectively, the x, y, and z components of the body's velocity). In other words, the body's momentum p is the 3-vector formed by P's spatial components. The temporal component of P is $m\gamma c$, which is E/c. In Euclidean geometry, the square of the hypotenuse's length is, of course, the sum of the squares of the other two sides' lengths. But in space-time geometry, the square of a 4-vector's length is the square of its temporal component's length *minus* the square of the length of the 3-vector formed by its spatial components. Accordingly, the length of the 4-vector P is $\sqrt{[(E/c)^2 - p^2]}$, which (as we just saw) equals the invariant mc. In short, P's length is an invariant quantity, though its decomposition into energy and momentum components is frame dependent, just as the electromagnetic field appears as different combinations of electric and magnetic fields from the standpoints of different inertial frames. Relativity thus unifies energy and momentum in exactly the same sense as it unifies the electric and magnetic fields.[15]

Classically, a body's momentum $p = mv$, whereas relativistically, $p = m\gamma v$. By calling a body's $m\gamma$ its "relativistic mass" M, momentum in relativity remains mass times velocity, and so an analogy can be drawn between relativistic and classical physics. Whereas m is invariant, M is not, since γ is obviously frame dependent in being a function of v. One version of our favorite equation, $E = Mc^2$, then follows from $E = m\gamma c^2$. The celebrated "$E = mc^2$," featuring the body's (invariant) mass m, holds only when the body is at rest (that is, when its $p = 0$)[16] since $E = mc^2$ then follows from the more general

$$\sqrt{[(E/c)^2 - p^2]} = mc$$

When the body is at rest ($v = 0$), its $\gamma = 1$, and so its "relativistic mass" $m\gamma$ equals its mass m. That is why "m" is sometimes called the body's

[15] That p is conserved in every inertial frame entails that E is so conserved as well, since p in one frame is a function of p and E in another. But this entailment alone does not suffice to unify momentum and energy—does not reveal them to be nothing more than aspects of the same single real thing—any more than momentum, energy, and mass are unified in classical physics by the fact that momentum conservation follows from energy conservation and mass conservation in every inertial frame.

[16] Thus the question posed in footnote 13 is answered: $E = mc^2$ holds only for E in the $p = 0$ frame.

"rest mass": its "relativistic mass" when it is at rest. Because the so-called "relativistic mass" is not an invariant quantity (and the term 'rest mass' refers back to 'relativistic mass'), the best thing to do in order to avoid confusing frame-dependent quantities with invariant ones is just to avoid the terms 'relativistic mass' and 'rest mass', and instead to stick solely with 'mass' for the invariant quantity symbolized m.[17]

Now we can ask a question regarding the proper interpretation of $E = mc^2$: Given that mass is a real property (since it is Lorentz invariant) whereas energy is not, how can mass and energy be the same thing ("mass-energy") *in the same sense* that distance in miles measures the same real thing as distance in feet? Moreover, in what sense can mass be *converted* into energy (as when a tritium nucleus decays) when mass and energy are not on a par in terms of their reality? We shall look into this.

On the basis of $E = mc^2$ (when $p = 0$), Einstein[18] says that "[t]he mass of a body is a measure of its energy content" (in the $p = 0$ frame). But beware: this remark may mislead us by suggesting that energy content is a basic, objective fact whereas mass is merely some sort of indicator, symptom, or manifestation of energy. In fact, however, mass rather than energy is the Lorentz invariant, and hence real quantity.[19] Accordingly, I cannot agree with remarks like Bertrand Russell's[20]:

> [W]e must abandon attempts to say what energy is. We must say simply: there is something quantitative, to which we give the name "energy"; this something is very unevenly distributed in space; there are small regions in which there is a great deal of it, which we call "atoms", and are those in which, according to older conceptions, there was matter.... Mass is only a form of energy.... It is energy, not matter, that is fundamental in physics (*ibid.*, pp. 27, 291).

[17] That Einstein did not regard "rest mass" and "relativistic mass" as standing on the same ontological footing is evident from his many remarks urging others to introduce as "mass" only a quantity independent of motion. See Arthur Fine, "Appendix" to John Earman, "Against Indeterminacy," this JOURNAL, LXXIV, 9 (September 1977): 535-38, p. 538; and Lev Okun, "The Concept of Mass," *Physics Today*, XLII, 6 (June 1989): 31-36.

[18] "Does the Inertia of a Body Depend Upon Its Energy Content?" in *The Collected Papers of Albert Einstein, Volume 2*, pp. 172-74, see p. 174. See also his *Relativity: The Special and the General Theory* (New York: Crown, 1931), pp. 55-56.

[19] Of course, there is an objective fact regarding energy content *in the $p = 0$ frame*, but this is a cheap sort of objectivity: it holds in all frames because it refers to a specific frame.

[20] *Human Knowledge: Its Scope and Limits* (New York: Simon and Schuster, 1948).

How can mass really be nothing but concentrated energy[21] that moves through space, "a form of bound energy,"[22] when mass rather than energy is the invariant quantity? Because energy is not Lorentz invariant, it seems difficult to endorse these remarks from Werner Heisenberg[23]:

> Since mass and energy are, according to the theory of relativity, essentially the same concepts, we may say that all elementary particles consist of energy. This could be interpreted as defining energy as the primary substance of the world (*ibid.*, p. 67).

This kind of talk filters down to monographs for guiding physics teachers, such as one by Robert M. Cotts and Robert W. Detenbeck,[24] which says, "Energy appears in still another wonderful disguise as mass itself" (*ibid.*, p. 66).

Here is a final issue regarding the ontological significance of Einstein's famous equation. In any inertial frame, a closed system's total energy and total momentum (as defined relativistically) are each "conserved." That is, in any inertial frame, the system's total energy does not change as time passes, and likewise for its total momentum. (But these totals differ in different inertial frames; total energy and total momentum are frame dependent and therefore not real.) Since E and p are conserved and we just saw an equation by which m is determined by E and p, a closed system's total m must be conserved in any inertial frame. But we saw earlier that when mass "turns into" energy (as in the decay of a tritium nucleus), there is a "mass defect" and so, apparently, mass fails to be conserved. (In these cases, textbooks commonly refer to some "missing mass" Δm.) We must look into how mass conservation can be reconciled with "mass defects."

II. WHAT IS MASS?

In classical physics, a body's mass is often interpreted as the amount of some "stuff" (matter) of which the body is made. In relativity, however, this interpretation cannot be correct. That is because properties that represent the quantity of some "substance" must obey the following principle: the total quantity of some sort of stuff in a system is the sum of the various quantities of that stuff belonging to the

[21] Hermann Weyl, *Space, Time, Matter* (New York: Dover, 1922), p. 200.

[22] Hans Pagels, *The Cosmic Code* (New York: Simon and Schuster, 1982), p. 37.

[23] *Physics and Philosophy* (London: George Allen and Unwin, 1958). The difficulties for understanding energy as a locally well-defined, conserved quantity become even more substantial as we pass to the general theory of relativity. See Carl Hoefer, "Energy Conservation in GTR," *Studies in History and Philosophy of Modern Physics* 31B, 2 (June 2000): 187-99.

[24] *Matter in Motion* (Seattle: Washington UP, 1966).

system's parts (where those parts are finite in number, nonoverlapping, and together exhaust the system).[25] For example, since the density of a whole is not the sum of the densities of its parts, density does not measure the amount of some stuff, and similarly for temperature and velocity. On the other hand, mass in classical physics is "additive" in this way. But in relativity, mass is not additive.

To see this, let us work in an inertial frame in which $p = 0$. (There always exists such a frame, and since mass is invariant, the result in that frame is applicable to any other inertial frame.) Then (as I have already mentioned)

$$E = mc^2$$

Consider a system of finitely many constituents, where each exerts on the others only negligible forces when they are not in contact—such as a gas consisting of many molecules.[26] Since the system's total energy is the sum of the energies E_1, E_2,...of its constituents, we have

$$m = (1/c^2)[E_1 + E_2 + \ldots]$$

Recall that for any constituent (say, the ith one),

$$E_i = m_i\gamma_i c^2 = m_i c^2/\surd[1 - (v_i^2/c^2)]$$

Now for any x where $x^2 < 1$,

$$1/\surd[1 - x] = 1 + (1/2)x + [(1 \cdot 3)/(2 \cdot 4)]x^2 + [(1 \cdot 3 \cdot 5)/(2 \cdot 4 \cdot 6)]x^3 + \ldots$$

Let x be (v_i^2/c^2). If v_i is low compared to c, then the terms with x^2 and higher powers of x are negligible, and so we can use the approximation

$$E_i \approx m_i c^2 + (1/2)m_i v_i^2$$

Notice that this is just $m_i c^2$ plus the ith constituent's classical kinetic energy. Substituting for E_1, E_2,...

$$m \approx (1/c^2)[m_1 c^2 + (1/2)m_1 v_1^2 + m_2 c^2 + (1/2)m_2 v_2^2 + \ldots]$$

and so

[25] See Norman Robert Campbell, *Foundations of Science* (New York: Dover, 1957), pp. 282-83, 286 [originally *Physics: The Elements* (New York: Cambridge, 1920)]. For related discussion, see Helen Morris Cartwright, "Amounts and Measures of Amount," *Noûs*, IX, 2 (May 1975): 143-64, pp. 147-49.

[26] This restriction means that there is no potential energy, there are no fields, and so forth; in classical terms, the only energy is kinetic energy. I relax this restriction in chapter 8, section 5 of my *Locality, Fields, Energy, and Mass* and go on to examine the mass possessed by the electromagnetic field.

$$m \approx [m_1 + m_2 + \ldots] + (1/c^2)[(1/2)m_1v_1^2 + (1/2)m_2v_2^2 + \ldots]$$

Hence, the system's mass m exceeds the sum of its constituents' masses. (They differ by an amount that reflects the constituents' kinetic energies in the frame where the system has zero total momentum. This will shortly be important.) So we cannot think of a body's mass as the amount of matter that forms it, where matter is a "stuff."

Note that the system's mass *is* the sum of its constituents' *relativistic* masses $m_i\gamma_i$ in the frame where $p = 0$. That is because in that frame,

$$m = (1/c^2)[E_1 + E_2 + \ldots] = (1/c^2)[m_1\gamma_1c^2 + m_2\gamma_2c^2 + \ldots]$$

$$= m_1\gamma_1 + m_2\gamma_2 + \ldots$$

But since a body's relativistic mass is frame dependent, it is not the body's quantity of any sort of real "stuff." So the fact that the system's (invariant) mass is the sum of its constituents' relativistic masses in the $p = 0$ frame does not show that the system's mass is its total quantity of some sort of "stuff" of which it is made.

Bearing in mind that mass is not additive, consider the following argument, some version of which appears in most relativity texts. Suppose two bodies, each of mass m_0, crash into each other and stick together, forming one body at rest of mass m_1. In the words of one fine text by Robert Resnick and David Halliday[27]:

> [Although] the total energy E of the system of particles will...be conserved [and] relativistic mass is conserved in this collision, *rest mass* [what I am calling simply "mass"] is not; m_1, the rest mass after the collision, is greater than $2m_0$, the rest mass before the collision.... After the collision, no kinetic energy remains. In place of the "lost" kinetic energy, there appears internal (thermal) energy, recognizable by the rise in temperature of the colliding particles.... [By $E = mc^2$,] (increase in thermal energy) = (increase in rest mass)(c^2). Thus we see that the decrease in kinetic energy for this isolated system is balanced by a corresponding increase in mass energy... (*ibid.*, p. 110).

So the text says that although the system is "isolated," mass is not conserved! This is not correct. In this argument, the sum $2m_0$ of the masses before the collision is shown to be unequal to the mass m_1 of the body formed in the collision. But $2m_0$ is not "the rest mass before the collision" (that is, the mass of the system before the collision) because *mass is not additive*. The argument that mass is not conserved rests on the mistaken assumption of mass's additivity. To say that

[27] *Basic Concepts in Relativity and Early Quantum Theory* (New York: Wiley, 1985, 2d ed.). I have substituted 'm_1' for their symbol for the resultant body's mass.

energy is conserved whereas mass is not helps to make energy seem like some sort of real stuff, leading to remarks like Heisenberg's (already quoted). Of course, it is also simply confusing for the text I have quoted to say on one page that the isolated system's rest mass is not conserved and on the very next page that "if we consider a closed system...then we may regard the rest mass of the body (or of the system) as constant."

If a body's mass is not its total quantity of some sort of stuff of which it is made, what is a body's mass? A body's mass is the property it possesses which determines the acceleration it undergoes in response to a force: $F = p'$ (as in classical physics), where in relativity, $p = m\gamma v$ and p' is the rate at which p is changing. When referring to a body's "mass," then, we must be thinking of that body as a thing that can feel a force and respond to it (by moving) *as a unit.* Nevertheless, no macroscopic body is elementary; any macroscopic body is also a system of bodies. Its motion, then, is nothing but the motions of its constituents, and these motions are determined by *their* masses and the forces that *they* feel. The remarkable fact is that the law of nature by which the constituents' motions are determined by their masses and the forces they feel is *the same as* the law by which the macroscopic body's motion is determined by its mass and the forces it feels: $F = p' = (m\gamma v)'$. In other words, the law "scales up."[28]

Let us see precisely why this fact is so remarkable. Macroscopic bodies are presumed to have elementary constituents (that is, constituents having no constituents themselves) and there is a law relating the force felt by an elementary constituent, the constituent's motion in response to that force, and a *single parameter* characterizing the constituent. This need not have been; we can imagine a universe in which a constituent's motion in response to forces is determined in different ways by several of its properties (such as its shape, volume, and chemical activity) rather than there being a single parameter (its "mass") that captures everything about the constituent affecting its motion in response to a force. But from the fact that such a parameter exists for elementary bodies, it does not follow that a single parameter characterizing the macroscopic body is all that is needed to allow its motion to be related to the force that it experiences. If a body's "mass" is defined as that single parameter, then even given the remarkable fact that the fundamental constituents of macroscopic

[28] A similar remark appears in Richard Feynman, Robert Leighton, and Matthew Sands, *The Feynman Lectures on Physics, Volume 1* (Reading, MA: Addison-Wesley, 1963), p. 19-2. For related discussion, see David Bohm, *The Special Theory of Relativity* (New York: Routledge, 1996), pp. 82-85, 110-18.

bodies have masses, it is a remarkable fact that macroscopic bodies *have masses too.*

Furthermore, from the fact that macroscopic bodies have constituents with masses *and* that macroscopic bodies have masses too, it does *not* follow that the law by which the elementary constituents' motions are determined by their masses and the forces they feel is *the same as* the law by which the macroscopic body's motion is determined by its mass and the forces it feels. If a body's "mass" is defined as the quantity with which it is associated that plugs into the particular law relating an *elementary* body's motion to the forces it feels, then once again, it is a remarkable fact that macroscopic bodies *have masses too.*

In short, neither of these conceptions of what mass *is* requires that macroscopic bodies have masses, even considering that their elementary constituents do. Furthermore, given either of these conceptions of mass *and* that a macroscopic body has a mass, there is nothing inevitable about its mass being the sum of its elementary constituents' masses (as is the case classically but not relativistically). So these conceptions of mass should inoculate us against the temptation to think that a body's "mass" *is defined as* the quantity of matter composing it.

It is certainly convenient for us that in order to predict a macroscopic body's motion in response to a force exerted upon it from outside it (an "external force"), we do not need to determine how each of its constituents responds to the various forces exerted upon it by other constituents ("internal forces"). We can simply think of the macroscopic body's mass m as if it were concentrated in a point particle located at the body's "center of mass" and then determine this particle's response to the external force. It is also convenient that the law governing this response is the same as the law governing the responses of the body's elementary constituents to the forces they feel. Otherwise, our investigations into the behavior of macroscopic bodies would not necessarily have left us well equipped to understand the behavior of their elementary constituents.

Let us see exactly why in classical physics, Newton's second law ($F = ma$, where F is a force felt by a body of mass m and a is the body's acceleration in response to that force) "scales up" in that it governs the motions not only of the elementary bodies, but also of the center of mass of a system of those bodies. To simplify the discussion, suppose there are only three elementary bodies, each exerting forces on the others. Let m_i be the ith body's mass, and let a_{ij} be the component of the ith body's acceleration that is caused by the force F_{ij} exerted by the jth body on the ith body. By Newton's second law,

$F_{ij} = m_i\, a_{ij}$. By Newton's third law ("Every action has an equal and opposite reaction"), $F_{ij} = -F_{ji}$, and so $m_i\, a_{ij} = -m_j\, a_{ji}$. Now consider bodies 2 and 3 as forming a single system of bodies. Let us see why that system's motion is likewise governed by Newton's second law. The system's center of mass is a kind of average of the positions of the system's constituents. Each constituent's contribution to the average is proportional to its mass. So the system's center of mass is a *weighted* average of its constituents' positions—weighted by their masses. If x_i is the position of the *i*th body, then the position x of the center of mass of the system composed of bodies 2 and 3 is given by

$$x = [m_2 x_2 + m_3 x_3]/[m_2 + m_3]$$

For Newton's second law to govern this system's motion would be for the force exerted on the system (namely, the sum of the forces exerted on the system's constituents by body 1, the body outside the system: $F_{21} + F_{31}$) to equal the system's mass ($m_2 + m_3$; in classical physics, mass is additive) multiplied by the acceleration a of its center of mass. From the above equation for x, it follows that

$$a = [m_2 a_2 + m_3 a_3]/[m_2 + m_3]$$

So Newton's second law governs the system's motion if and only if

$$F_{21} + F_{31} = (m_2 + m_3)\, a = (m_2 + m_3)[m_2 a_2 + m_3 a_3]/[m_2 + m_3]$$
$$= m_2 a_2 + m_3 a_3$$

By Newton's second law applied to the constituents,

$$m_2 a_2 = F_{21} + F_{23}$$
$$m_3 a_3 = F_{31} + F_{32}$$

So Newton's second law governs the system's motion if and only if

$$F_{21} + F_{31} = F_{21} + F_{23} + F_{31} + F_{32}$$

But this is obviously true since (by Newton's third law)

$$F_{23} = -F_{32}$$

So the system consisting of bodies 2 and 3 behaves in relation to body 1 in just the way that one elementary body behaves in relation to another. (By expanding the system to encompass more and more elementary bodies, we could work up to the conclusion that macroscopic bodies obey Newton's second law.) Thus, it takes the cooperation of Newton's third law for Newton's second law to scale up. Since either or both these laws could have been different, there is nothing logically mandatory about the fact that the law relating a body's motion to the forces it feels scales up.

It is likewise the case in relativity theory that a collection of bodies behaves as a single body with a mass m located at the collection's center of mass. According to relativity, the system's center of mass is the weighted average of its constituents' positions—weighted by their energies (or, what comes to the same thing, their relativistic masses):

$$x = [m_1\gamma_1 x_1 + m_2\gamma_2 x_2 + \ldots]/[m_1\gamma_1 + m_2\gamma_2 + \ldots]$$

(This formula is restricted to the case where the constituents exert forces on one another only when they are in contact—in other words, when no fields are involved. At the start of this section, I restricted myself to this case for the sake of simplicity.) For example, if the system is closed (that is, feels no external forces), then (since it behaves as a single body) its center of mass undergoes no acceleration. Notice that the weights in the weighted average are frame-dependent quantities (since the ith weight involves γ_i, which is a function of the ith body's velocity, which is obviously frame dependent). So the location of the system's center of mass is not Lorentz invariant.

Now we are in a position to appreciate the key point.

III. THE CONVERSION OF MASS TO ENERGY IS NOT A PHYSICAL PROCESS (OR VICE VERSA)

The frame in which the system's total $p = 0$ is the frame in which the system's center of mass is at rest. As we have already seen, the system's mass increases as the kinetic energies of its constituents increase in the $p = 0$ frame—that is, as its constituents move about more quickly relative to the system's center of mass. That is because in this frame, the system's

$$m = E/c^2 = (1/c^2)[E_1 + E_2 + \ldots]$$

and so any increase ΔE in the constituents' total kinetic energy contributes $\Delta E / c^2$ to the system's mass, though it has no effect on any constituent's mass. Imagine, for example, a ball of hot gas, its molecules whizzing around randomly. We add to the constituents' total kinetic energy by heating the gas. We thereby increase the gas's mass. In what sense is this the "conversion" of energy into mass?

Suppose that initially we think of the gas as a collection of molecules, each with its own mass. In other words, we treat each of these bodies as individually feeling forces and being accelerated by them, its acceleration depending on its mass. Accordingly, we characterize the heat as having boosted various molecules' kinetic energies, but not their masses. Suppose we then change our perspective by considering the ball of gas as a single body that feels external forces and moves about as a unit. From this perspective, the kinetic energy contributed by the heat becomes part of the gas's mass. This "conversion" of energy into mass is not the transformation of one kind of stuff into another, since neither energy nor mass is a kind of stuff or measures the amount of some kind of stuff (energy because it is not Lorentz invariant, mass because it is not additive). But more importantly, we have just seen that this "conversion" of energy into mass is not any kind of real physical process taking place in nature. *We* "converted" energy into mass simply by *changing our perspective* on the gas: from treating it as many bodies to treating it as a single body.

Let us see that again. Suppose we begin by treating the gas as a single body. The body is heated. Heat energy flows into it and its mass increases by an equivalent amount. It looks like energy is being converted into mass; fluid, gossamer energy has "solidified" or "congealed" into matter, "the extended hard 'stuff' with which we are all familiar" (in Zahar's words). I argue, however, this is not a real process; rather, it is just an artifact of the perspective we have adopted. No such "conversion" occurs on a different perspective. Let us begin the gas's story again and this time, let us treat the gas as many bodies. We find no energy being transformed into matter as the heat is being added to the gas—so long as we continue to regard the gas as many bodies; none of those bodies increases its mass while the gas is heating up. The heat energy goes into their kinetic energies relative to the gas's center of mass.

So on the first perspective, energy was converted into mass, whereas on the second, no such conversion occurred. Furthermore, at whatever point in the story we choose to switch our perspective on the gas, we can make it appear that at that point, energy is being transformed into mass—even if at that point, heat is no longer being added to the gas! Again let us begin by treating the gas as many bodies, and let us maintain this perspective throughout the heating of the gas. Suppose the heat source is then shut off. Nothing more is now happening to the gas. But we decide at this point to shift our perspective in telling the gas's story; henceforward we treat the gas as a single body. In this shift, the gas molecules' total kinetic energy (in the $p = 0$ frame) contributes an equivalent amount to (and so "becomes part of") the gas's mass. But obviously, no physical change accompanies this "transformation"; again, nothing is happening to the gas. This "conversion" of energy into mass is not a physical process. Thus, whether and when a conversion of energy into matter occurs in the story of the gas depends on the perspective from which we elect to tell that story and any shifts of perspective we make in the course of telling it.

The distinction between forms of "internal" and "external" energy is often drawn by relativity texts.[29] But the point is seldom made that because this distinction is a scientific convention, so is the "conversion" of energy to mass or vice versa. As far as science is concerned, the line between bodies—between "internal" and "external"—is a convention (that is, not built into the universe, but rather drawn onto it by us) because of the remarkable fact that I emphasized in the previous section: that *any* system is characterized by a *mass* determining the way it responds (by moving as a unit) to the forces it feels as a whole (that is, to external forces). As Einstein[30] says: "Every system can be looked upon as a material point as long as we consider no processes other than changes in its translation velocity as a whole" (*ibid.*, p. 225).

A shift in "perspective" (*not* reference frame!), in what are being treated as single bodies, obviously takes place in the course of the textbook discussion (quoted earlier) of two bodies colliding and subsequently sticking together. The text treats the system as initially consisting of two bodies, each with its own mass. But the text regards the system after the collision as forming a single body. The "conversion" of energy into mass in this case is an artifact of this shift in perspective.

Sometimes, a shift in perspective is much subtler. Return to the tritium nucleus (one proton, two neutrons) that decays into a helium-3 nucleus (two protons, one neutron) and an electron and antineutrino that fly off at high speed. There is a "mass defect" in that the masses of a helium-3 nucleus, an electron, and an antineutrino add up to less than the mass of a tritium nucleus. The missing mass is said to have been "converted" to the kinetic energies of the resulting bodies. But this "conversion" of mass into energy is not real; it is an illusion produced by a subtle shift in our perspective. (The transformation of the tritium's neutron into a proton, an electron, and an antineutrino is, of course, a real occurrence.) We treated the system as initially forming a single body: a tritium nucleus. But we treated the system after the decay as consisting of three bodies, each with its own mass. The system's mass after the decay is the same as the system's mass before the decay. There is no "mass defect" here; mass is conserved. The "mass defect" appears to arise from the fact that the

[29] See, for example, Resnick and Halliday, pp. 111-12.

[30] "Elementary Derivation of the Equivalence of Mass and Energy," *Bulletin of the American Mathematical Society*, XLI, 4 (April 1935): 223-30.

sum of the three masses after the decay is less than the system's mass before the decay (the difference reflecting the three bodies' kinetic energies in the $p = 0$ frame). But the sum of the three masses after the decay is less than the *system's* mass *after* the decay. Mass is not additive, and our expectation that it is additive (arising because we expect it to measure the amount of some stuff) leads us to refer to a "mass defect"—to ask where the "missing mass" has gone and to conclude that it has turned into energy. The "mass defect" results not from some physical transformation of matter-stuff into energy-stuff, but rather from our illicitly trying to view the system from two different "perspectives" at the same time.

Thus, the conservation of mass does not conflict with the original tritium nucleus's mass exceeding the sum of the postdecay bodies' masses. The "mass defect" is not real; it is produced not by the decay of the tritium nucleus, but by our treating the postdecay system as a collection of bodies even while treating the predecay system as a single body. The fact that Δm of the system's initial mass "becomes" energy $(\Delta m) c^2$ when we think of the postdecay system as a collection of bodies, each with its own mass, does not mean that mass is really nothing but energy or that mass and energy are like distance in feet and in miles—different ways of measuring the same property. Again, the "conversion" of mass into energy occurs because we have shifted our perspective, not because the nucleus has decayed. Unlike the conversion of a caterpillar into a butterfly, the conversion of mass into energy (or vice versa) is not a physical process.

A body's energy $m\gamma c^2$ is its kinetic energy plus mc^2. Since this latter energy depends only on its mass, we can think of its mass as "associated with" a certain quantity of energy: a "mass energy" mc^2. Accordingly, it might sometimes help to think of relativity as having turned the classical law of energy conservation into the conservation of "mass-energy" by having added a term for the energy "frozen" as a body's mass. In Einstein's[31] words:

> Before the advent of relativity, physics recognised two conservation laws of fundamental importance, namely, the law of conservation of energy and the law of conservation of mass; these two fundamental laws appeared to be quite independent of each other. By means of the theory of relativity they have been united into one law (*ibid.*, p. 54).

But the fact that, in totaling a system's energy, we must include not only the more familiar forms of energy but also terms of the form mc^2

[31] *Relativity: The Special and the General Theory.*

does not mean that mass really *is* energy (or "massergy") or "that mass is now viewed as a form of energy"[32] like ice is a form of water. Mass is a real property whereas energy is not. Mass-energy is not some sort of real stuff that is neither created nor destroyed.

Just as there is only a single object, the electromagnetic field, which in different inertial frames appears as different combinations of electric and magnetic fields, so there is only a single real property, the body's mass, which in different frames appears as different combinations of its energy and momentum. This is the sense in which relativity theory unifies energy, momentum, and mass.

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[32] Marshall Spector, *Methodological Foundations of Relativistic Mechanics* (Notre Dame: University Press, 1972), p. 151.