Studies in History and Philosophy of Science 87 (2021) 44–53

Contents lists available at ScienceDirect

Studies in History and Philosophy of Science

journal homepage: www.elsevier.com/locate/shpsa

What could mathematics be for it to function in distinctively mathematical scientific explanations?

Marc Lange

Department of Philosophy, University of North Carolina at Chapel Hill, CB#3125, Chapel Hill, NC, 27312, USA

ARTICLE INFO

Article history:
Received 1 October 2019
Received in revised form
4 February 2021
Accepted 5 February 2021
Available online 13 March 2021

1. Introduction

Several philosophers have suggested that some scientific explanations work not by virtue of describing aspects of the world's causal history and relations, but rather by citing mathematical facts. This paper investigates what mathematical facts could be in order for them to figure in such "distinctively mathematical" scientific explanations.

For example, Lange (2013) has suggested that Mother fails in her attempt to divide 23 strawberries evenly among 3 children because 23 is not divisible evenly by 3. Baker (2005) has suggested that the fact that 13 is a prime number explains why cicadas with 13-year life-cycles tend to suffer less predation than cicadas with 12- or 14-year cycles. Pincock (2007) has suggested that Euler's theorem explains why no one ever traversed each of the bridges in the Königsberg arrangement exactly once. Some philosophers (e.g., Baker, 2009) regard such explanations as supplying an "enhanced indispensability argument" for platonism by analogy with arguments for scientific realism: we ought to believe in the existence of any entities (including abstract mathematical objects) we believe indispensable for explaining physical facts.

As an alternative to platonism, representationalism – a.k.a. the "indexing" account (Melia 2000:473; Daly & Langford, 2009) – holds that we use mathematical terms to refer to some physical property when, for example, we use some number of units to express that physical property. For instance, we use number vocabulary to "index" a distance relation when we say that two objects are 1.459 centimeters apart. When the objects' separation partly explains their mutual gravitational attraction, no number – no such abstract entity – figures among the explainers. The explainers are instantiations of physical properties (Melia 2000:473). But having established a mapping between physical properties and mathematics, we can use math to infer from certain physical facts to others. For instance, we can use a mapping to refer mathematically to two objects' masses and separation, use $F = GmM/r^2$ to calculate F, and finally (by mapping from math back to physical properties) arrive at the objects' gravitational attraction. Representationalism says that every scientific application of mathematics so operates – even in "distinctively mathematical" explanations.

For this mapping procedure to succeed empirically, the relations among various possible physical facts must mirror the relations among various mathematical claims. For instance, the various possible mass facts that we pick out as something's being 1 g, being 2 g, and so forth must stand in relations having the same structure as the mathematical claims that something equals 1, equals 2, and so forth. (For instance, masses and numbers must both be ordered by "greater than" in the same way.) Perhaps the mirroring must in some respects be richer than an isomorphism (Bueno & Colyvan, 2011), or perhaps the mirroring can be looser (Balaguer, 1998). What the "morphism" (as Baron, Colyvan, & Ripley (2017:9) term it) must be need not concern us. The point of "representationalism" is that mathematics functions in science as a mere "representational device" (Balaguer, 1998:139). By virtue of a morphism between mathematical structures and physical systems, we use math to represent various physical properties, manipulate those representations, and map the mathematical result back to physical claims, arriving at scientifically useful information. In this way, math might even be indispensable for science; we may have no other practical means of so referring and inferring. But (representationalism insists) we can so use mathematics without regarding it as consisting of true claims about abstract mathematical entities, just as scientific anti-realism maintains that scientists need not regard terms like "electron" and "electric field" as successfully referring to concrete unobservable entities in order to use those terms in inferring from past observations to other observable facts.

Representationalism, then, is a view about how mathematical terms operate *in science*; in this paper, our particular concern will be with how they operate in "distinctively mathematical" scientific explanations. A rival view of how mathematical terms operate in

E-mail address: mlange@email.unc.edu.

https://doi.org/10.1016/j.shpsa.2021.02.002

science is a platonist view, according to which mathematical terms in science describe abstract mathematical entities existing outside of spacetime and having no causal relations. This platonist view of mathematical language *in science* would naturally be coupled with the same view of how mathematical language operates *in mathematics*. After all, if mathematical abstracta must exist for mathematics to play its *scientific* roles, then those abstracta are available to be what claims *in mathematics* concern.

By contrast, although representationalism (as a view about mathematical language *in science*) could be combined with some form of nominalism about how mathematical language operates *in mathematics*, representationalism could instead be combined with platonism about how mathematical language operates in mathematics. That is, one might hold that mathematical terms as *science* employs them pick out physical properties, whereas mathematical terms in *mathematics* refer to mathematical abstracta. Since this paper's main concern is the scientific application of mathematics (especially in "distinctively mathematical" scientific explanations), by "platonism" I will generally mean a view about how mathematical language operates *in science*.

Representationalism can take many forms. On one form, mathematical "claims" (at least as science employs them) are not claims at all; they are uninterpreted, empty formalism lacking truth-value. An alternative form of representationalism takes mathematics as consisting (at least for scientific purposes) of claims about certain uninterpreted formal systems – that is, as claims about what follows from the axioms of a given system. These claims have truth-values. On another form of representationalism, mathematical claims (at least for scientific purposes) purport to describe mathematical abstracta, but science does not involve believing in the truth of mathematical claims. Leng's (2010, 2012) representationalism goes even further by adopting fictionalism, according to which science takes mathematical claims to be false, the mathematical abstracta posited by these claims not to exist. On this view, science engages in games of making believe that mathematical abstracta exist and stand in morphisms to certain physical facts. Various results about numbers at which we arrive while "making believe" are mapped to physical facts. Mathematical claims are "true in the story of mathematics" in the way that "Sherlock Holmes lives at 221B Baker Street" is true in Conan Doyle's stories. (On Leng's view, not only *science* but also *mathematics* engages in games of making believe that mathematical abstracta exist.)

Instead of adopting representationalism and so being like scientific anti-realists, we could be like scientific realists by adopting platonism. However, despite platonism's differences from representationalism, platonism (I will argue) invokes the same sort of mapping procedure to account for math's applicability to science (as emphasized by Balaguer, 1998:137 and Bueno & Colyvan, 2011:346, 366-7). Where representationalism invokes morphisms between mathematical formalisms (or fictions) and aspects of the physical world, platonism invokes morphisms between mathematical abstracta and aspects of the physical world. For instance, a morphism between mass properties and the real numbers, considered as abstract entities, enables us to draw accurate conclusions about massive objects by mapping their masses to numbers, reasoning mathematically, and then mapping back to the physical world.

I will argue that because of this similarity between representationalism and platonism, the reason that representationalism cannot take "distinctively mathematical explanations" as genuine explanations of physical facts by mathematical facts carries over to platonism. The enhanced indispensability argument aims to support platonism by appealing to the view that in "distinctively mathematical explanations," mathematical facts explain physical facts. However (I will argue), platonism precludes mathematical facts from playing that explanatory role for the same reason as representationalism precludes mathematics from doing so. I will then present one account of what mathematical facts are that allows them to be explainers in distinctively mathematical explanations.

These arguments will presuppose an account of how distinctively mathematical explanations work. In section 2, I will sketch that account (versions of which have been proposed by several philosophers), according to which distinctively mathematical explanations are not a *sui generis* variety of explanation. Rather, they work just like certain other non-causal scientific explanations, such as an explanation of the fact that gravitational and electromagnetic forces are alike in conserving energy. That explanation appeals to the fact that the law of energy conservation *constrains* the kinds of forces there could have been. Like this explanation, a distinctively mathematical scientific explanation is an "explanation by constraint." That other scientific explanations can be understood as "explanations by constraint" supplies independent motivation for this account of distinctively mathematical explanations.

In section 3, I will elaborate why representationalism precludes mathematics from supplying constraints on physical facts. Then I will argue that platonism, too, must deny that distinctively mathematical explanations are "explanations by constraint". Accordingly, if we regard distinctively mathematical explanations as explanations by constraint, then we cannot accept representationalism or platonism. What can we accept (or do we have here a good reason against interpreting distinctively mathematical explanations as explanations by constraint)? In section 4, I will identify an interpretation of mathematical facts already in the literature, "Aristotelian realism", under which mathematical facts can serve as constraints on physical facts. Aristotelian realism is one way that mathematical facts could be in order for them to function as explainers in distinctively mathematical explanations.

This paper's strategy is to presuppose an account of distinctively mathematical explanations as a certain kind of non-causal explanation (as "explanations by constraint") and then to use that account to investigate what mathematics could be in order for it to play this explanatory role. An alternative strategy, which (as we will see) others have employed, is to presuppose an account (such as representationalism) of what mathematics is, at least for scientific purposes, and then to use that account to investigate how "distinctively mathematical explanations" must work. Neither approach can be judged before it has been tried. But this paper's approach has gone largely untried because it requires beginning with an account of some non-causal scientific explanations, and the topic of non-causal explanations has (until recently) been relatively neglected; during the past few decades, research on scientific explanation has been dominated by accounts of causal explanation. Accordingly, this paper's approach has at least the virtue of novelty.

The enhanced indispensability argument aims to draw an analogy between belief in mathematical abstracta and belief in various unobservable concrete entities. This analogy is disrupted by the fact that these concrete entities purportedly figure in *causal* explanations, whereas mathematical abstracta are not putative causes. I will now sketch an account of how distinctively mathematical explanations explain according to which they are like certain other *non-causal* scientific explanations. To use that analogy to understand what mathematics could be is the original approach taken by this paper.

2. Explanations by constraint

The law of energy conservation says roughly that for any finite region of space and finite period of time, the change in energy inside the region equals the energy that has crossed its boundary during that period. Energy conservation follows from the various force laws (such as the laws of gravitation and electrostatics) and other dynamical laws (including that this inventory of force laws contains all of the force laws there are). But is this derivation explanatory? If so, the force laws are explanatorily prior to energy conservation; the fact that gravitational forces conserve energy is due partly to the gravitational force law, and the fact that electrostatic forces conserve energy is due partly to the electrostatic force law. It is a coincidence that both forces conserve energy.

Alternatively, that both forces conserve energy is no coincidence if energy conservation *constrains* the kinds of forces there could have been so that only forces that conserve energy are possible. Then energy conservation is explanatorily prior to the force laws rather than the other way around. The conservation law is the common reason why both gravitational and electrostatic forces conserve energy. Lange (2017) calls this an "explanation by constraint", and I will follow that approach here. Huneman (2018) and Morrison (2018:210) characterize other laws, such as spacetime symmetry principles,[1] as also serving as constraints in non-causal explanations.

The force laws possess a variety of necessity: "natural necessity". Their inventory deems certain hypothetical fundamental forces to be possible and others (e.g., an inverse-cube gravitational force) impossible. If the conservation laws of energy, momentum, etc. are constraints rather than coincidences, then they carve out a broader species of possibility; certain hypothetical kinds of forces covered by none of the actual force laws (and hence naturally impossible) nevertheless could have existed because they would have conserved these quantities. Energy is then conserved not as an upshot of the dynamical laws, but in a way that "transcends" (Wigner 1972:13) the particular dynamical laws there happen to be. Energy conservation then possesses a stronger variety of necessity than the force laws do. This is the crucial similarity between energy conservation and mathematical facts (e.g., that 23 is not evenly divisible by 3): though energy conservation's necessity (as a constraint) is not as strong as mathematical necessity, it is stronger than ordinary natural necessity, just as mathematical facts possess a variety of necessity stronger than natural necessity. They can constrain, too.

Lange (2017) distinguishes "constraints" from "coincidences" in terms of counterfactual conditionals. If energy conservation is a coincidence of the force laws, then energy conservation might not still have held, had the force laws been different. The conservation law depends on the force laws. By contrast, if energy conservation is a constraint, then energy conservation would still have held, even if the force laws had been different. As a constraint, energy conservation is not dependent on the particular force laws there happen to be. Rather, it limits the force laws there could have been. Mathematical facts are also constraints: 23 would still have been indivisible evenly by 3, even if the force laws had been different.

On this picture, an "explanation by constraint" explains by providing information about how the explanandum is required by constraints on causal processes and hence is necessary in a stronger way than causal powers could render it. If energy conservation is a constraint, then a phenomenon it explains is not due to any causal factors, but rather is inevitable considering the framework within which any cause *must* act. This "must" involves a stronger variety of necessity than ordinary natural necessity. A "distinctively mathematical explanation" is (I propose) just an "explanation by constraint" where the constraint is a mathematical fact. Mother's attempts to distribute 23 strawberries evenly among 3 children would still have failed even if she had used different strawberry distribution mechanisms – indeed, even naturally impossible ones. This distinctively mathematical explanation does not explain as causal explanations do (namely, by supplying information about causal relations), but rather by revealing Mother's failure to have been more inevitable than the inventory of causal powers could render it.

By regarding distinctively mathematical explanations as "explanations by constraint," we avoid treating them in an *ad hoc* fashion. The explanation of Mother's failure proceeds in the same way as energy conservation's explanation of our failure to do certain things. For instance (Lange, 2017:51), we have never been able to discover or create an incompressible, non-viscous, homogeneous fluid that begins to circulate when placed at rest feeling no external forces (besides a uniformly downward pull). We have failed because such a fluid *cannot* begin to circulate, since doing so would violate energy conservation – just as Mother failed because her task was impossible.

Lange (2009, 2017) draws the analogy between mathematical facts and constraining conservation laws by positing a hierarchy of grades of necessity. Necessity is intuitively associated with inevitability, unavoidability – maximal persistence under counterfactual antecedents. Accordingly, each variety of necessity is possessed by a set of truths that is "stable" in that the set's members possess maximal counterfactual invariance: they *would* all still have held under every counterfactual antecedent under which they *could* (i.e., without contradiction) all still have held. More precisely (Lange says), a set of truths qualifies as "stable" exactly when it is deductively closed and in every conversational context, its members would still have obtained under any counterfactual antecedent that is logically consistent with each of them. The mathematical truths (together with the logical truths) form one stable set, the addition of the conservation laws (if the conservation laws are constraints) forms another, and the addition of the rest of the natural laws forms another.[2] There is a 1-1 correspondence between the grades of necessity and the stable sets (that are "non-maximal," i.e., that do not include every fact). Lange (2009: 37-8; 2017:78) proves that the stable sets must form a nested hierarchy: for any two stable sets, one must be a proper subset of the other. A smaller stable set's members, having a broader range of counterfactual invariance associated with their stability, possess a stronger variety of necessity.

This account identifies what makes the mathematical facts like constraining conservation laws, allowing them to play analogous explanatory roles. The counterfactuals associated with their status as constraints (namely, that both the conservation laws and the mathematical facts would still have held, had the force laws been different) are among the counterfactuals that render the corresponding sets "stable" and that are associated with their members' especially strong varieties of necessity. Let's now see what mathematical facts could be (or cannot be) in order for "distinctively mathematical explanations" to be explanations by constraint.

[1] The spacetime symmetry principles include, for example, that the laws are invariant under arbitrary spatial and temporal displacements. Einstein's principle of relativity is another spacetime symmetry principle. Spacetime symmetries are often thought to non-causally explain the great conservation laws and the special relativistic length contractions and time dilations. These would be "explanations by constraint". That spacetime symmetries are constraints rather than coincidences is Penrose's point in characterizing Einstein's insight as "that one should take relativity as a *principle* rather than as a seemingly accidental consequence of other laws" (1987:24) and Wigner's point in saying that these symmetries are not "based on the existence of specific types of interaction" (1964:958).

[2] Perhaps there are stable sets that include some but not all of the mathematical truths. Lange (2009:78) considers this briefly. All that this paper needs is that the mathematical truths (together with the logical truths) form one stable set that omits the natural laws.

3. Against representationalism and platonism

According to representationalism, mathematics functions as a device for representing physical facts. On this view, I will now argue, mathematics does not supply its own facts that (perhaps in combination with physical facts) explain facts about the physical world.

On one version of representationalism (recalling section 1), mathematical "claims" have no truth-values because they are uninterpreted formalism. Since they do not state facts, they cannot state facts that (partly) explain why some physical fact obtains. (Of course, mathematics can nevertheless figure in an explanation in the same way as it figures in other scientific work: by assisting us in picking out a physical fact and inferring from it. But none of that involves mathematical facts explaining.) On other versions of representationalism, mathematical claims have truth-values either because they are claims about what an uninterpreted formal system's axioms entail or because they are false but true "in the story of mathematics." Nevertheless, on either of these alternatives, mathematics fails to supply its own facts that (partly) explain physical facts because neither a fact about an uninterpreted formalism nor a fact in a fiction concerns the physical world. Neither what happens in a fiction nor what happens "in" an uninterpreted formalism can be even partly responsible for what happens in the physical world; in particular, neither can constrain the physical world.

The reason that representationalism cannot regard mathematical facts as constraints is that representationalism takes mathematics as functioning in science solely to refer to physical properties, and this role does not give mathematics itself the power to constrain what it refers to. That is, representationalism takes mathematics to operate in science as a "map", and maps (while *telling us* about what they map) do not help to *explain why* what they map is as it is. A city map, for instance, does not help to explain why the city's street grid is what it is. Rather, the street grid helps to determine what a map must be like in order to represent that grid accurately. The features of a map do not impose constraints (in section 2's sense) on the features of the world being mapped, whether by "the features of a map" we mean the features themselves (which have no truth-value) *or* truths about the map *or* what is true "in the story of the map". A city map does not have the power to make the city conform to it. According to representationalism, some bit of mathematics is either suitable or not for representing some aspect of the world (and this function exhausts its scientific role), but that bit of mathematics does not make the world be such that the mathematics is suitable for representing it.[3]

Representationalists themselves generally recognize and embrace this argument and its conclusion that mathematics cannot explain physical facts. For instance, Strevens (2018:115) endorses representationalism and holds that for mathematics to constrain physical facts is "incompatible with representationalism, on which mathematics has no power to constrain what laws there can be. (The representationalist holds that our representations of the laws must conform to mathematical principles because the principles are built into our system of representation, not because they are built into the world.)" It is *our representations* of the world that must conform to math, rather than *the world* being constrained by mathematical facts having greater necessity than the ordinary causal laws. Accordingly, Strevens denies that mathematical facts (even partly) explain physical facts. For instance, Strevens (2018:100) holds that Euler's theorem does not (partly) explain why no one has ever crossed the Königsberg arrangement of bridges. Rather, the explaining is done by some non-mathematical fact about the bridges that is represented mathematically: "There is something non-mathematical about the bridges of Königsberg that renders them untraversable; that non-mathematical fact is represented by Euler's theorem and so – eliding; conflating; metonymizing – we say that the failure of any attempt at traversal is 'because of' Euler's theorem itself."

My approach (recalling section 1) is the opposite of Strevens's. He begins with an account of what mathematics is (for the purposes of science) – representationalism – and from that account he draws conclusions about whether mathematics explains physical facts. Conversely, I begin from a picture of how mathematics explains physical facts (namely, by constraining them in the same way as many scientists have regarded conservation laws as doing). From that picture, I aim to infer what mathematics could be (for the purposes of science). Thus, whereas Strevens concludes from representationalism that mathematics does not explain physical facts, I reject representationalism as failing to do justice to distinctively mathematical explanations.[4]

Here is another way to capture the idea that mathematics, as merely a representational device, is too detached from the physical world to (partly) explain facts about it. I suggest that for fact F to (partly) explain fact G, nothing can pose an in principle bar to its being the case that had F not obtained, then G would (or, at least, might) not have obtained. That is, if it is in principle impossible for a difference in F's holding to make a difference to G's holding, then F cannot (partly) explain G.[5] For instance, consider causal explanation: there is no in principle bar to the counterfactual dependence of one event on another or on a law of nature. Of course, a given effect may not in fact be counterfactually dependent on its cause because in the absence of the actual cause, a backup would have stepped in to cause the effect. Nevertheless, there might have been no backup, so it is not in principle impossible for the effect to

[3] Saatsi (2011:146) likewise argues for the explanatory impotence of mathematics under representationalism on the grounds that maps, although allowing us to infer to various features of what they map, do not explain why what they map is the way it is: "In as far as Euclidean geometry in mathematics correctly represents the structure of physical space, we can use the former to infer facts about the latter, just as we can use an accurate map to infer facts about the surrounding topography, say. But facts about physical space are not thereby explained by mathematics, any more than facts about surrounding topography are explained by the map."

[4] Bueno and Colyvan (2011) make a similar point (without appealing to "explanation by constraint"): the mapping account (as they term "representationalism") cannot allow mathematics to explain physical facts since "it is hard to see how a mere representational system can provide explanations and yet that is the only role mathematics is allowed to play in the mapping account" (p. 351). "According to the mapping account, mathematics is a mere representational tool, and any explanation that drops out of the mathematics must be just standing proxy for the real physical explanation. … The street map does not explain – facts about the city do" (p. 371). But unlike Strevens, Bueno and Colyvan want to leave room for mathematics to explain. They believe their enriched version of the mapping account (which they term the "inferentialist conception" of the application of mathematics) does so. I do not see how their approach does better, since (as Batterman (2010:9) emphasizes) it sees mathematics as serving science only through mappings.

[5] I am not proposing that for F to (partly) explain G, it is necessary (or sufficient) that had F not obtained, G would not have obtained. Baron (2016:372) seemingly maintains that if F figures in G's explanation, then the truth of "Had ∼F, then ∼G" is necessary and sufficient for F to be serving not merely as a representational device, but as positing real entities. This is similar to my proposal but not the same. My proposal does not demand G's counterfactual dependence on F, but merely that nothing in principle preclude G's counterfactual dependence on F. My proposal presents this condition as necessary (rather than necessary and sufficient, if F figures in G's explanation) for F to play more than a representational role. Baron's apparent proposal seems too strong since F can (partly) explain G (not serving merely as a representational device) without "Had ∼F then ∼G" holding, as when a backup to F would have caused G had ∼F. My proposal allows this, as long as there is nothing in principle precluding G's counterfactual dependence on F (e.g., precluding the absence of a backup). (Note that the counterfactual antecedent is simply "Had ∼F"; it does not posit some more specific alternative to F.)

counterfactually depend on its cause. Thus, the above "counterfactual test" poses no obstacle to a causal explanation – for instance, to the masses of the bodies placed at the ends of the two arms of a see-saw, the lengths of those arms, and Archimedes' law of the lever (and various background conditions) explaining why the see-saw tips to one side (rather than balancing or tipping to the other side).

Crucially, this counterfactual test also applies to non-causal explanation – e.g., to "explanation by constraint." Suppose that the energy conservation law is a constraint, possessing stronger necessity than a given force law. Had energy conservation not been a law, it would not have been there to constrain the various forces to conserve energy, so energy might not have been conserved by some interaction. Thus, the counterfactual test does not preclude energy conservation (as a constraint) from explaining why the various force laws conserve energy. However, if energy conservation is a constraint, then the counterfactual test precludes the force laws from (partly) explaining energy conservation. As a constraint (in section 2's sense), energy conservation's greater modal strength than the force laws possess ensures that energy would still have been conserved, even if the force laws had been different. Energy conservation's counterfactual dependence on a force law is in principle impossible when energy conservation is a constraint. So if energy conservation is a constraint, the counterfactual test rightly precludes the constraint's being (partly) explained by what it constrains.

How does the counterfactual test apply to a mathematical fact's "explaining by constraint"? Consider the conditional: Had 23 been divisible evenly by 3, then Mother would have successfully divided her 23 strawberries evenly among her 3 children.[6] I regard this conditional as non-trivially true.[7] So the counterfactual test does not preclude the fact that 23 is divisible evenly by 3 from explaining why Mother fails to divide her 23 strawberries evenly among her 3 children.

But representationalism does not allow this conditional to be non-trivially true, and so (as I argued earlier) representationalism deems mathematics unable to (partly) explain physical facts. According to representationalism, mathematics is scientifically useful because of a morphism between the mathematical apparatus and physical facts. Had the mathematical apparatus been different, the morphism would not have held; that is, had the apparatus been different, then it would no longer have been well-suited to representing the physical world. The physical world would have been no different. After all, had a city map depicted the street grid differently from the way it actually does, then the grid being mapped would have been no different (since the map has no power to make the city conform to it). The map would simply not have been useful; it would have been inaccurate. Accordingly, take a form of representationalism that construes "Had 23 been divisible evenly by 3" as "Had it been 'true in the story of mathematics' that 23 is divisible evenly by 3." (Or take a form of representationalism that construes this counterfactual antecedent as positing that the given formalism's axioms entail that 23 is divisible evenly by 3.) Such a view must say that had 23 been divisible evenly by 3, then the mathematical apparatus would not have been useful (as least as science actually employs it); making believe that the "story of mathematics" is true (or using the formalism to represent physical facts) would not have been empirically successful.[8]

Baron et al. (2017) provide a useful comparison with (and contrast to) my argument above. For mathematics to (partly) explain some physical fact, my "counterfactual test" does not require any counterfactual conditional to be true, but requires merely that there be no in principle bar to its being the case that had the mathematics doing the explaining been different, then the physical explanandum would have been different. Baron et al. require that such a conditional obtain for the corresponding explanation to hold; they see these conditionals as "implicated in extra-mathematical explanations" (p. 6), i.e., in "the explanation of physical facts (in part) by mathematical facts" (p. 1). For example, they maintain that for the cicada fact to be explained by facts about primality, it must be the case that "If, in addition to 13 and 1, 13 had the factors 2 and 6, North American periodical cicadas would not have 13-year life cycles" since having prime life cycles is the optimal way to avoid predation by predators with periodical life cycles and 13 would not then have been prime (p. 6). Baron et al. endorse both this explanation and such counterfactuals. I agree with them that such conditionals require that under the conditional's antecedent, the morphism would still have held:

> …[I]n order for changes within mathematics to ramify into the physical world, there must be some link between the mathematical and physical structures such that a change to the mathematical structure implies a corresponding change in the physical structure. This link must be held fixed as part of the counterfactual supposition in order for the mathematical twiddle to properly ramify. The link that we propose to hold fixed is a morphism. The broad idea is that mathematical systems and physical systems share particular structural features. … [H]olding fixed the fact that the mathematical structure maps the physical structure is enough to allow twiddles to the mathematics to ramify across to the physical system at issue. (p. 9)

6 Some philosophers (e.g., Jansson and Saatsi (forthcoming)) have proposed counterfactual accounts of non-causal explanations such as the strawberry example. However, these philosophers generally focus on counterfactuals such as "Had there been 24 strawberries rather than 23, then Mother would have been able to distribute them evenly," rather than antecedents that posit countermathematicals ("Had 23 been divisible by 3…"). Also Mother's failure may counterfactually depend on a mathematical fact that does not help explain her failure: Had 20 been divisible by 3, then 23 would have been divisible by 3 and so Mother would have been able to distribute them evenly.

7 Of course, this conditional's antecedent posits a mathematical impossibility. Although standard counterfactual semantics deems all countermathematicals to be vacuously (i.e., trivially) true because there are no possible worlds where the antecedent holds (Lewis, 1986:111), some philosophers have recently argued that there are non-trivially true countermathematicals and have proposed semantics accommodating them (e.g., Baron (2016), Baron et al. (2017:18), Jenny (2018), and references therein). Other philosophers (e.g., Schaffer, 2016) presuppose that such counterfactuals are non-trivial without offering any semantics. Mathematicians often treat countermathematicals as non-trivial. (For example, Gardner (1960:154) writes, "If pi could be expressed as a rational fraction or as the root of a first- or second-degree equation, then it would be possible, with compass and straightedge, to construct a straight line exactly equal to the circumference of a circle.")Some readers will not countenance regarding counterfactuals such as "Had there been finitely many numbers, then there would have been finitely many prime numbers" and "Had there been finitely many prime numbers, then there would have been a largest prime" as non-trivially true. To these readers, I say: The non-triviality of these countermathematicals is presupposed in this paper only in using the "counterfactual test" as an additional argument that representationalism and platonism cannot (whereas Aristotelian realism can) account for the role that mathematical truths play as "constraints" in distinctively mathematical explanations. You may stick to the other arguments I give for this conclusion – arguments that do not appeal to countermathematicals' non-triviality.

8 The counterfactual "Had 23 been divisible evenly by 3, then Mother would have succeeded in dividing 23 strawberries evenly among 3 children" fails even more quickly on other versions of representationalism. If "23 is divisible evenly by 3" is empty formalism, then the antecedent is meaningless. If the antecedent is understood as "Had it been false that 23 is not divisible evenly by 3", then according to fictionalism, this antecedent is true and so the conditional is false since Mother actually fails to divide her strawberries evenly.

Baron et al. aim to specify the “general procedure” by which we “evaluate” the conditionals that (they maintain) must be true for mathematics to explain physical facts (p. 6). They stipulate that the procedure “holds fixed” (p. 6) the morphism under the counterfactual antecedent. In this way, the procedure deems these conditionals true.

However (I say), for mathematics to explain a physical fact, the morphism should not have to be held fixed *by stipulation*. This comes too close simply to stipulating that mathematics is able to explain physical facts and that the physical facts are counterfactually dependent on the mathematics. The “mathematical twiddle” must ramify to physical facts not because of some *ad hoc* procedure for evaluating the conditional, but because there is an appropriate connection between the mathematics and the physical facts – a kind of connection that is invariant under the counterfactual antecedent. If that connection is supposed to be a morphism, then there should be a reason why that morphism is in fact invariant – a reason that applies in the same way to some other facts, determining whether or not they also are invariant. The morphism's invariance should not have had to be inserted expressly “by hand” into the procedure for evaluating the counterfactuals. Under representationalism (as I have just argued), the morphism is not invariant except under an artificial procedure stipulated as holding the morphism fixed.[9]

Baron et al. might reply that if platonism instead of representationalism turns out to be required for mathematical facts to explain physical facts, then (instead of remaining officially neutral on the existence of mathematical abstracta) Baron et al. would declare for platonism. But (I will now suggest) platonism faces exactly the same problem as representationalism does. Mathematical abstracta, even if they exist, would be too detached from the physical world to help explain facts about it. The realm of platonic entities is like the world of the story of mathematics. Just as what is true-in-the-story-of-mathematics cannot constrain the physical world, so likewise what is true in the platonic realm cannot constrain the physical world.

This argument can be expressed in terms of the “counterfactual test.” Suppose platonism: the fact that 23 is not divisible by 3 is a fact about mathematical abstracta. This fact's standing apart from the physical world poses a bar in principle to its being the case that had this fact not obtained, then Mother would (or might) have been able to distribute her 23 strawberries evenly among her 3 children. Mother would still have failed even if mathematical abstracta had not existed or even if they had still existed but stood in different relations.[10]

Admittedly, platonism takes mathematical abstracta as standing in *some* relations to physical entities. For example, on *ante rem* structuralism, an arrangement of physical objects, such as a given baseball team's defensive arrangement on a given occasion, can “exemplify” a certain abstract structure (Shapiro, 1997:99–100). But had that abstract structure possessed a different property, then the team's defense on the given occasion would not have exemplified it. The arrangement of the players on the field would have been no different; it would simply not have fit that abstractum. This is precisely analogous to the case with representationalism: had the representation been different, it would no longer have stood in a morphism to a given aspect of the physical world, and likewise had the given *ante rem* structure been different, it would no longer have been exemplified by a given aspect of the physical world.[11] Structuralists emphasize that “applying a branch of mathematics to a given target domain depends upon the target domain having a structure homomorphic to a structure treated by the mathematics being applied… We can then use ordinary mathematics to derive properties of this representing structure, and know that, under the representing homomorphism, they transfer to the target domain in the guise of synthetic descriptions” (Resnik, 1992:119, 122). Because morphisms play similar mediating roles in both platonism and representationalism, these two views are vulnerable to the same objection: neither can account for mathematics' explanatory role in science because the morphism would have broken down under the key counterfactual antecedent.

My point is a generalization of the challenges to platonism famously issued by Benacerraf (1973) and Field (1989). Benacerraf argued that it is mysterious how we could know about mathematical abstracta because we are not in causal contact with them. By comparison, I am arguing that it mysterious how mathematical abstracta could constrain (and thereby explain) physical facts when

[9] Parallel argument: Suppose that the match's remaining unlit is explained partly by its remaining unstruck. Suppose that this explanation is bound up with the truth of “Had the match been struck, then it would have lit.” Then the fact (required by this conditional's truth) that the match would still have been surrounded by oxygen, had it been struck, must be true *not by stipulation*, but rather because the match's being oxygenated *earns* its truth under that counterfactual antecedent in the same way as the match's being lit earns its truth under this antecedent. This is, after all, the motivation for using possible worlds to resolve Goodman's (1947) cotenability problem.

[10] Baker (2003) lists philosophers holding that physical explananda would have been no different, had mathematical abstracta been different or nonexistent. Armstrong (1978:68) joins them: “Is it not clear that *a*'s whiteness is not determined by *a*'s relationship with a transcendent entity? Perform the usual thought-experiment and consider *a* without the Form of Whiteness. It seems obvious that *a* might still be white” (cf. Field, 1989:238). Baker (2003) and Baron (2016:377) see the “intuition” that a physical explanandum would have been no different, had mathematical abstracta been different or nonexistent, as begging the question against platonism. I do not see this as a mere intuition; I have argued for it, by analogy with facts about what is true “in the story of mathematics” (or what an uninterpreted formal system's axioms entail): Had the math fact been different, it's the morphism (rather than the physical explanandum) that would have been different, according to views (whether representationalist or platonist) employing morphisms to connect math facts to physical facts. A platonist who accepts that a physical explanandum would have been different, had some mathematical fact been different, cannot simply take platonism as underwriting this counterfactual dependence. The platonist must show how platonism can account for the physical explanandum's counterfactual dependence on mathematical facts.In any case, we do not have to appeal to the “counterfactual test”, but only to the account of mathematical explanations as explanations by constraint, to insist that the platonist who regards mathematical facts as explaining physical facts must account for how facts about mathematical abstracta can “constrain” physical facts. Insofar as that account posits a novel kind of constraining (that is, unlike the way that symmetry principles and conservation laws constrain other laws), platonism is rendered less plausible. That is my main point here.Mary Leng suggested to me that platonism needs no morphism; Hume's principle suffices. I think Hume's principle supplies a morphism and (here comes the counterfactual) for 23 not to have been divisible by 3, Hume's principle would have to have broken down.

[11] A similar argument applies to “heavy duty platonism” (Knowles, 2015), according to which physical magnitudes are fundamental relations between physical entities and numbers (as mathematical abstracta). On this view, numbers do not index physical properties, but rather help to constitute them. (For instance, being 10 kg just is bearing the mass-in-kg relation to 10.) No morphism needs to be added for numbers to apply to physical objects because a morphism has, in effect, already been baked into a given physical property; that property is a relation to a number. Therefore, had numbers been different (e.g., had 23 been divisible by 3), the properties that they help to constitute would have been different – in which case the mass property partly constituted by a given number would (or, at least, might) not still have been instantiated by the physical objects that actually instantiate the mass property partly constituted by that number. (This is analogous to the fact that had a given abstract structure possessed a different property, then the team's defense that actually exemplifies the given structure would – or, at least, might – not still have exemplified it.) For instance, had 10 been larger, the given object (that is actually 10 kg) would not have borne the mass-in-kg relation to 10. Likewise, had 23 been divisible by 3, Mother's aggregate of strawberries would no longer have borne the number-of-strawberries relation to 23, but it would still have been impossible for Mother to distribute her strawberries evenly among her children. The counterfactual test for mathematics to explain by constraint is therefore not passed.

there is no causal, nomological, constitutive, or similar connection between them. Field's challenge is that "it appears in principle impossible to explain" (p. 26) a correlation between facts about mathematical abstracta and our beliefs about mathematical abstracta (and that this in principle impossibility tends to undermine belief in them). By the same token, the correlation between facts about mathematical abstracta (such as that 23 is indivisible evenly by 3) and natural facts (that 23 strawberries are never divided evenly into 3 groups) seems equally mysterious. This "in principle impossibility" is reflected in the counterfactual test, which sees the mathematical abstracta's standing outside the physical world as in principle barring its being the case that had the mathematical abstracta 23 and 3 been different, then Mother would (or might) have fared differently.[12]

Symmetry principles (e.g., that the first-order laws are collectively invariant under an arbitrary spacetime displacement, i.e., under the replacement of s and t with $s+\Delta s$ and $t+\Delta t$) concern first-order laws of nature.[13] So they are suited to constraining the first-order laws (and they do so, if they have the right invariance under counterfactual antecedents). The same applies to conservation laws constraining dynamical laws. But it is difficult to see how facts about mathematical abstracta, even if they had the right invariance under counterfactual antecedents, could constrain facts about physical objects (i.e., could pass the counterfactual test). Both facts about mathematical abstracta and facts about representational schemes require morphisms to allow them to bear upon physical facts. These morphisms lack the modal strength required for mathematical facts to constrain; the morphisms would have broken under the key counterfactual antecedent in the counterfactual test.

Of course, we could reject "explanation by constraint" and try to find a different conception of how mathematical facts explain, at the risk of making their explanatory power *sui generis* rather than of the same kind as symmetry principles and conservation laws possess. Alternatively, we could find an account of what mathematics is about (for the purposes of science) that requires no independent morphism for mathematics to apply to the physical world and thereby allows mathematics to constrain physical facts. Let's try that.

4. For Aristotelian realism

According to "Aristotelian realism" (Bigelow, 1988; Forrest & Armstrong, 1987; Franklin 2008, 2014), mathematics concerns mathematical properties possessed by physical systems. Since mathematics directly concerns the physical world rather than representational schemes or mathematical abstracta, there is no obstacle to mathematical facts constraining that world. No morphism is needed to connect mathematics to the physical world, so the "counterfactual test" is passed: there is no morphism to fail under the key counterfactual antecedent. Mathematical truths are truths about the physical world and some of them possess an especially strong variety of necessity. Accordingly, they can constrain the physical world.[14]

To appreciate how mathematics explains under Aristotelian realism, consider how the version of Aristotelian realism about natural numbers given by Forrest and Armstrong's (1987) applies to Mother distributing her strawberries. The aggregate of strawberries stands contingently in a certain relation to the unit-making universal of being a strawberry. That relation, according to Forrest and Armstrong, is the number 23. Likewise, the aggregate of children stands contingently in a relation (3) to being a child. Those two relations (which, of course, are also instantiated by many other pairs of aggregates and unit-making universals) stand necessarily in a certain relation: indivisibility. That relation's holding is the fact that 23 is not evenly divisible by 3. That fact helps to explain Mother's failure to distribute her strawberries evenly (without cutting any) among her children. What is it about the aggregate of strawberries that prevents their being divided evenly among the children? Plainly, it is their standing in the 23 relation to being a strawberry. Appropriately, an aggregate of 23 jellybeans stands in the same relation to being a jellybean – and its doing so is what prevents its being divided evenly (without cutting any jellybeans) among the children. It is no coincidence that neither is divisible evenly among the children; they have something else in common that is responsible for this similarity. Likewise, what is it about the strawberries' standing in the 23 relation that makes their doing so preclude their being divided evenly among the children? Plainly, it is that the 23 relation stands necessarily in a certain relation (indivisibility) to 3. (Appropriately, this is the same relation in which 24 stands to 5, for instance.) In this way, the strongly necessary fact that 23 is indivisible by 3 can constrain Mother just as conservation laws can constrain the force laws.[15]

Furthermore, the counterfactual test is passed. Just as Mother might have succeeded in distributing her strawberries evenly had she possessed different numbers of strawberries or children, so likewise she might have succeeded had 23 not stood in the relation of indivisibility to 3, since the strawberries' standing in the 23 relation to strawberryhood would then no longer have precluded their being divided evenly among 3 children. No morphism mediates between numbers and physical objects; the numbers are relations in which physical aggregates stand. So there is no opportunity for the morphism to break down, rather than the physical explanandum be different, under the key counterfactual antecedent. The intuitively plausible conditionals in the counterfactual test are thus vindicated.

Aristotelian realism thereby allows us to say what various philosophers want to say about mathematical explanations but what their official accounts (I have argued) preclude. For example, regarding a mathematical explanation of our failure to carry out a given construction with compass and straightedge, Leng (2012:990) says: "the explanation is explanatory because it shows an empirical phenomenon (our inability to find certain ruler/compass constructions) to result from the general structural features of the situation. It enables us to see…the phenomenon as resulting from a necessity not just from bad luck or poor construction … The explanation is explanatory … because it shows a

[12] Some platonists (e.g., Brown, 2012) agree that mathematical abstracta cannot explain.

[13] See note 1.

[14] "Aristotelian realism" permits uninstantiated properties; as Franklin (2008:105) says, this is a "Platonist or modal Aristotelianism", not a "strictly this-worldly Aristotelianism". It remains Aristotelian because some mathematical properties are literally possessed by physical things, even if it is contingent which of these properties are instantiated. Some mathematical truths about mathematical properties are necessary; they would hold whether or not the properties are instantiated. Also note that Aristotelian realism does not suggest that some mathematical truth is of interest only insofar as it has scientific utility.

[15] Rosen (2016:15) calls Forrest and Armstrong's view "eccentric". But Rosen describes their view as "that numbers and the rest are (despite appearances) concrete entities". On the contrary, Forrest and Armstrong identify numbers not with concrete entities, but rather with relations in which concrete entities stand to unit-making universals. Likewise, Pincock (2013:271) says that on Franklin's view, "mathematical entities are themselves part of the physical world" (2013:271) and "an inventory of the genuine constituents of the physical world reveals that these constituents include mathematical entities" (2012:4). Again, numbers on Franklin's view are properties, not concrete entities. (See also note 14.)

number of particular empirical phenomena to result from purely structural features of the problem situation." I thoroughly agree. But Leng says that fictionalism can account for this explanation because we can deduce results in Euclidean geometry and then "transfer" those results to physical space though an "interpretation" of physical space as approximately Euclidean. (This "interpretation" is a morphism.) But although this procedure enables conclusions about physical space to be *deduced*, I have maintained that a fictional entity's possessing certain properties in its fiction cannot *explain why* we are unable to carry out a construction.[16] A fiction cannot constrain, and I have taken math to explain by constraining. Had the "Euclidean story" contained entities possessing different properties, the construction in physical space would still have been impossible, but Euclidean geometry would not have been a reliable means of deducing facts about physical space; the "interpretation" of physical space as approximately Euclidean would not have worked. To underwrite her understanding of how this mathematical explanation operates, Leng needs physical space (or objects therein, or relations among them) to possess (at least approximately) some property that mathematics describes as standing in various necessary relations. Only in that way can an empirical phenomenon result from "general structural features of the situation" – the *physical* situation, having features described by mathematics.

Aristotelian realism characterizes mathematics as concerning mathematical properties possessed by physical systems and the necessary relations in which those properties stand. Physical systems are obviously the subjects of the examples that (we saw at the outset) philosophers have proposed of scientific explanations that work by citing mathematical facts (rather than by describing aspects of the world's causal history and relations). Therefore, Aristotelian realism is especially well-suited to dealing with the properties that tend to figure in these putative scientific explanations. Consider, for example, the proposed explanation of why no one ever managed to traverse a path crossing all of the bridges in the Königsberg arrangement exactly once. The explanation is that a necessary and sufficient conditions for a network's traversability is that it have zero or two nodes having an odd number of edges touching them (Euler's theorem) and that Königsberg's bridges have four nodes of odd degree, so it is impossible to traverse them, so no one did. As Franklin (2008:113) emphasizes, an Aristotelian realist regards Euler's theorem as a mathematical truth "both necessary and about reality" in supplying necessary and sufficient conditions for a physical system's traversability. Being non-traversable and having all four nodes of an odd degree are both properties of the bridge arrangement itself.

Aristotelian realism sidesteps a problem encountered by the enhanced indispensability argument for platonism. Critics of that argument have pointed out that claims involving particular natural numbers can be replaced in (e.g.) the strawberry explanation by their "nominalized equivalents" using first-order logic with identity. For instance, "there are 3 children" can be replaced by

$$\exists x \exists y \exists z (Cx \& Cy \& Cz \& \, x \neq y \, \& \, x \neq z \, \& \, y \neq z \, \& \, \forall w (Cw \rightarrow w = x \vee w = y \vee w = z))$$

(where C is the property of being a child). That 23 is indivisible by 3 likewise has a nominalized equivalent. These equivalents may be cumbersome, but they explain Mother's failure without invoking mathematical abstracta, so those abstracta are not indispensable for explanations. In defending the indispensability argument, Baker (2017) replies that although math is dispensable for *deducing* Mother's failure, it remains indispensable for *explaining why* she fails, since an explanation in mathematical terms possesses virtues (generality, topic-neutrality, and unification) favoring it over its rival without mathematics. Knowles and Saatsi (ms.) reply that even if these virtues require mathematics, these virtues are purely pragmatic (in making the explanation more transparent to us) and so do not confirm that mathematical abstracta exist. I am not convinced that these virtues are purely pragmatic; perhaps they instead contribute to making the proposed explanation more plausible (as defenders of inference to the best explanation would insist). In that case, it is not obvious how to weigh the loss of parsimony brought by positing mathematical abstracta against the gain in these explanatory virtues, so as to evaluate which of the two rivals is, all things considered, the more plausible explanation.

Aristotelian realism sidesteps this problem because it raises no question about whether a candidate explanation positing additional entities (such as mathematical abstracta), though less parsimonious than its nominalized rival, repays this ontological investment with additional explanatory virtues, rendering it more plausible than its rival. Aristotelian realism posits no new mathematical abstracta: on Forrest and Armstrong's version, the fact that the aggregate of children stands in the 3 relation to the unit-making universal of being a child is not less parsimonious than its "nominalized equivalent", since neither of them posits mathematical abstracta. Of course, Aristotelian realism posits a relation between the unit-making universal and the aggregate of children. But the nominalized equivalent already invokes the property C of being a child (and no one denies that there is an aggregate of children). The "nominalized equivalent" simply expresses what it is for that aggregate to stand in the 3 relation to the property of being a child.[17]

According to Aristotelian realism, there is no competition between the nominalized explanation and the mathematical explanation; they are not rivals. An explanation invoking the fact that 23 is indivisible by 3 names a relation in which the 3 relation and 23 relation necessarily stand, constraining aggregates standing in those two relations (to respective unit-making universals). Knowles and Saatsi (ms.) consider whether Mother's failure could be explained without invoking 23's indivisibility by 3 – by instead appealing to the fact that if there are 23 objects (expressed in nominalized terms), then it is impossible to divide them evenly into 3 sections (in nominalized terms). Aristotelian realism sees this "nominalized equivalent" as expressing what it is for the 3 relation and 23 relation to stand in the indivisibility relation, so this explanation is no rival to an explanation invoking the fact that 23 is indivisible by 3. Note that the "nominalized equivalent" still invokes the *impossibility* of dividing the objects evenly. That impossibility must transcend the causal powers there happen to be (it must be some stronger sort of impossibility) for this fact to explain Mother's failure, since Mother's failure is independent of the particular causal interactions involved in distributing the

[16] Of course, a fictional entity could help us to *communicate* an explanation, as when we say that an electric field is like an invisible fluid between charges. But a fictional fluid cannot be responsible for an actual force. Likewise, we could use a fictional entity to pick out a structural property (possessed by the fictional entity in its story) that some physical entity possesses. But even if we find the fictional entity indispensable for picking out the structural property, nothing about the fiction is responsible for a physical fact.

[17] This paper makes no attempt to address *en passant* the general problem of how to understand properties. On Aristotelian realism, mathematical properties are thought to pose no problems beyond the problems that other properties of physical systems already pose. So an account of how other, ordinary physical properties operate (e.g., in scientific explanations) should apply as well to mathematical properties (which are also properties of physical systems).

strawberries. Any mathematically possible method of distributing strawberries, even one that is physically impossible (or involves non-spatiotemporal entities), would be constrained in the same way. This stronger modality is what powers distinctively mathematical explanations (when Aristotelian realism is combined with an account of distinctively mathematical explanations as "explanations by constraint").

The details of Forrest and Armstrong's proposal do not matter for my argument; perhaps a different version of Aristotelian realism would give a better account of natural numbers. But as long as natural numbers constitute properties (including relations) that aggregates of physical objects can possess (or stand in), there is no metaphysical gulf between facts about natural numbers and the physical facts targeted by distinctively mathematical explanations. Of course, facts about natural numbers are not the only mathematical facts figuring in distinctively mathematical explanations. But Aristotelian realism aims to construe these other mathematical facts as concerning properties of properties of…properties of natural numbers. If successful, Aristotelian realism can avoid the obstacles encountered by representationalism and platonism in accounting for the roles that these mathematical facts play in distinctively mathematical scientific explanations.[18]

Consider, for instance, the intermediate value theorem, which Colyvan (1998:321–2) cites as partly explaining why there are, at every moment in earth's history, two antipodal equatorial points at equal temperature. As long as every property figuring in the intermediate value theorem constitutes a property of a property of…a property of natural numbers, then since natural numbers are relations in which aggregates of physical objects stand, no metaphysical gulf must be crossed for the intermediate value theorem to partly explain some fact about physical objects. That is, there is no in principle obstacle to its being the case that had the intermediate value theorem not held, there might not have been (at every moment during earth's history) two antipodal equatorial points at equal temperature.

My aim is to show how Aristotelian realism avoids those obstacles, not to argue that Aristotelian realism is the only view that does so[19] — and certainly not to give a full-blown defense of Aristotelian realism. My main point in bringing up Aristotelian realism is to argue that there is at least one account of how mathematics operates in science that avoids the difficulties for other accounts that we saw in previous sections; if we could identify no account that holds the promise of avoiding those difficulties, then we would have some reason to believe that our understanding of these alleged difficulties is mistaken.

However, it is noteworthy that on Aristotelian realism together with my account of distinctively mathematical explanations as "explanations by constraint", mathematical facts playing no role in the scientific explanation of any physical fact still have to obtain in order for the mathematical facts that *do* figure in some distinctively mathematical explanations to possess the necessity empowering them to play their explanatory roles. For instance, suppose that N is a mathematical fact figuring in no scientific explanation. Perhaps N is an equation giving the sum of two natural numbers so ridiculously large that they figure in no scientific explanation. Let's now show that even though N figures in no scientific explanations, N must obtain in order for the fact that 23 is indivisible evenly by 3 to serve as a constraint in scientific explanations. Indeed, N's truth must possess the same necessity as the fact that 23 is indivisible evenly by 3; that is, N must also be a constraint.

In order for the fact that 23 is indivisible evenly by 3 to be a constraint, it must (according to section 2) belong to a (non-maximal) stable set. That is, the fact must belong to a deductively closed (non-maximal) set of truths the members of which collectively possess maximal counterfactual invariance: in every conversational context, its members would still have obtained under any counterfactual antecedent that is logically consistent with each of them. Suppose for the sake of *reductio* that N does not belong to any such set and so does not possess the same necessity as the fact that 23 is indivisible evenly by 3. Then for a given such set to be stable, it must be the case that in every conversational context, 23 would still have been indivisible evenly by 3 even if $\sim$N had been the case. But in some contexts, this conditional is not true; once one fact about mathematical sums, even a very large one, is posited to have been different, then (in some contexts) any other sum might also have been different. Hence, for the fact that 23 is indivisible evenly by 3 to belong to a non-maximal stable set and hence be a constraint, N must also be a constraint.

To help clarify this argument, let me give an analogous argument concerning natural necessity rather than mathematical necessity. Consider Coulomb's law giving the mutual electrostatic repulsion of any two point-charges. Suppose that as it happens, there never are any point-charges of exactly 1.23456789 statcoulombs. Had there been two such point-charges but their mutual electrostatic repulsion not conformed to Coulomb's law, then Coulomb's law would not have held. With Coulomb's law gone, there would have been no reason for other pairs of point-charges still to accord with Coulomb's law – just as had one addition equation been different, no other addition equation would have been safe. Hence, there is no non-maximal stable set containing the consequences of Coulomb's law for the actually instantiated values of electric charge but not containing the consequences of Coulomb's law for the values of electric charge that are not actually instantiated. In the same way, there is no non-maximal stable set containing the fact that 23 is indivisible evenly by 3 but not also containing N.

Thus, if Aristotelian realism can cover various mathematical facts involving properties never possessed by (aggregates of) physical objects, then even if those facts do not themselves figure in any distinctively mathematical explanations, they could still be required by the mathematical facts that do figure in those explanations.

5. Conclusion

I have argued that for "distinctively mathematical explanations" to be explanations by constraint, mathematical language cannot operate in science as representationalism or platonism describes. It can operate as Aristotelian realism describes. That is because Aristotelian realism enables mathematics to apply to the physical world without a morphism's mediation. Aristotelian realism portrays mathematical facts as concerning properties of physical entities (or properties of those properties, or…).

My argument has assumed that the role played by mathematical facts in distinctively mathematical explanations is like the role played (according to many scientists) by conservation laws (among

[18] Bigelow (1988), Forrest & Armstrong, 1987, and Franklin (2014) all examine how an Aristotelian realist account of natural numbers could be extended to other parts of mathematics: rational numbers, negative numbers, complex numbers, geometry, graph theory, operations research,…. I will not examine these proposals, since my purpose is not to give a full defense of Aristotelian realism, but merely to show that it holds promise for avoiding the obstacles (elaborated in previous sections) encountered by other accounts of mathematics' role in distinctively mathematical explanations (understood as "explanations by constraint").

[19] For example, on some elaborations of the view that mathematical facts are facts about what logically follows from given uninterpreted axioms, no morphism is needed. Rather, a mathematical fact is a narrowly logically necessary truth. Such necessities are constraints; they form a stable set. They explain by constraint. (Other elaborations are versions of representationalism, requiring a physical system to instantiate the axioms.)

other laws) as "constraints" on the dynamical laws. My approach is novel in not treating distinctively mathematical explanations as *sui generis*. Instead I have drawn upon an independently motivated account of an important variety of non-causal scientific explanation. This is an antecedently plausible model for distinctively mathematical explanations, considering that mathematical facts and conservation laws are both plausibly taken as possessing especially strong varieties of necessity and thus as transcending the world's causal network. That mathematical facts are more like conservation laws than like facts about electrons seems a promising avenue for future research.

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