Fuzzy Credence as Vague Credence: A Reply to Elga's Argument

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Abstract: Many philosophers hold that a rational person can have imprecise credences. A famous argument due to Adam Elga, however, purports to show that rationality requires that credences have precise values. I show that Elga's argument can be evaded if we understand imprecise credences to be a case of vagueness.

It is widely held that a rational person can have imprecise credences.[1] In the standard formalism, the doxastic state of such a person is represented not by a single probability function, but by a convex set of them, which may be called her *representor.* She is definitely committed to a certain probabilistic judgment – such as that A is more likely than B – just in case every function in her representor agrees in ratifying that judgment. There is a formal analogy here with the supervaluationist approach to vagueness[2], according to which every vague term is associated with a range of possible extension, which we may call the term's *allowable sharpenings*. A statement using a vague term is to be counted as definitely true iff it would come out true no matter which of its allowable sharpenings we treated as its extension. If it would come out as true relative to some of its allowable sharpenings and false relative to others, then it is a penumbral case, neither definitely true nor definitely false. One way of interpreting the standard formalism for imprecise credences is to take it as treating imprecise credences as vague credences, and adopting the supervaluationist approach to vagueness. For example, if Sarah has imprecise credences, then the term "Sarah's credence function" is vague, and its allowable sharpenings are exactly the probability functions found in Sarah's representor. On this interpretation of the standard formalism, some particular real number is Sarah's credence for the proposition A, though it might be indeterminate which number it is. On a different interpretation of this formalism, imprecise credences are not vague at all, though they might be interval-valued. Thus, Sarah's credence for A is precisely defined – but it might be precisely defined to be a certain interval of real numbers, rather than a single real number. You might think that nothing much could hang on the choice between these interpretations. But I aim to show here that if, and only if, we adopt the vagueness interpretation, we can easily get around a famous argument against imprecise credences due to Adam Elga.

[1] E.g. Joyce, J., "How Probabilities Reflect Evidence." *Philosophical Perspectives*, 19(2005), p. 171, and the references there cited.
[2] E.g. Fine, K., "Vagueness, Truth and Logic." *Synthese* 30 (1975).

Elga begins[3] by reasonably asking how rational imprecise credences, if such there be, are related to rational choice. Three obvious possibilities stand out:

> <u>Demanding:</u> S may rationally choose option A iff: for *every* function Pr in S's representor, A maximizes S's expected utility, calculated using Pr.
>
> <u>Strict:</u> S must somehow designate one function Pr in her representor; then, S may rationally choose A iff A maximizes S's expected utility, calculated using Pr.
>
> <u>Permissive</u>: S may rationally choose option A iff: for *some* function Pr in S's representor, A maximizes S's expected utility, calculated using Pr.

<u>Demanding</u> is too demanding: It's fairly obvious that in very many perfectly ordinary situations, it will imply that no matter what S chooses, she will choose irrationally. Elga argues persuasively that <u>Strict</u> is an unattractive view for a defender of vague credence.[4] In effect, it demands that we always act exactly as if we had precise credences, even if we don't really. But if a person's credences are in principle revealed in her dispositions to act, then it is then it is not clear that there is any real difference between obeying <u>Strict</u> and having precise credences.

This leaves <u>Permissive</u> – but Elga shows that it is subject to a clear counterexample.[5] Suppose that you have an imprecise credence for H, and that your representor includes some functions that give probabilities greater than 0.6, and some that give it probabilities less than 0.4. Now suppose that you will be offered two bets:

> Bet A: You win $15 if H; otherwise, you lose $10.
>
> Bet B: You win $15 if ~H; otherwise, you lose $10.

You must decide whether to take Bet A before you are offered Bet B. But you know in advance that you will be offered both.

One striking thing about this situation is that if you take both bets, you will certainly come out $5 ahead. By contrast, if you refuse both bets, you will break even. So if you decline both bets, you will have acted manifestly irrationally: Whether H is true or false, you could have come done better.

What does <u>Permissive</u> say about this case? There are some probability functions in your representor that give H a probability less that 0.4; on those, you maximize expectations if you decline Bet A. So you may rationally decline Bet A. But there are also functions in your representor that give H a probability greater than 0.6; on those, you maximize expectations if you decline Bet B. So, you may

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[3] Elga, A., "Subjective Probabilities Should Be Sharp." *Philosopher's Imprint*, 10(5), (2010), p. 3.

[4] Ibid, p. 6.

[5] Ibid, pp. 4-5.

rationally decline bet B. Thus, Permissive implies that you may rationally decline both bets. This is clearly wrong: You would have to be a fool to turn both of these bets down.

There is an obvious move to make here: Perhaps individual choices are not the only things that can be evaluated for rationality; perhaps sequences of choices can also be rational or irrational. And while there are probability functions in your representor on which you maximize expectations by refusing Bet A, as well as functions in it on which you maximize expectations by refusing Bet B, there are no probability functions in your representor on which the sequence consisting of refusing Bet A and then refusing Bet B maximizes your expectations. So the defender of Permissive might say that if you refuse both bets, then even though each of the individual choices you made perfectly rational, the sequence of choices you made was not – so you have still done something irrational.

A promising gambit. But Elga has a ready reply.[6] Consider two situations:

(I) You have already refused Bet A; now you are offered Bet B.

(II) You were never offered Bet A; now you are offered Bet B.

Notice that in these two situations, everything that seems relevant to the question of whether you should take Bet B is exactly the same: the options now open to you, the utilities of their consequences conditional on both H and ~H, and your doxastic attitude toward H. So rationally should require the same things of you in both situations. But on the sequences-proposal, it doesn't. In situation (II), you may rationally take or leave Bet B, as you like. But in situation (I), rationality requires you to take it. For to leave it would be to complete a rationally forbidden sequence of actions. Thus, the rule Permissive is still subject to counterexample, even if we interpret it as applying to sequences of choices as well as individual choices.

It seems to me that Elga's argument cannot be answered by anyone who believes that thinks that rational people can have imprecise credences, and who models the doxastic states of such people using the standard framework, interpreting it as attributing interval-valued credences to people. But things are not so bad if we opt for the vagueness interpretation instead. For if we do, we can answer Elga's question of how imprecise credences are related to rational choice very simply: In the very same way that precise credences are:

> S may rationally choose option A iff A maximizes S's expectations, calculated using S's credences.

Of course, if your credences are vague, then it might be vague what the expected payoff of some option is. And similarly, it might be a vague matter whether or not some option would maximize your expectations. And so, it can be a vague matter whether some option is rationally permissible for you or not. But so what? Vagueness is pervasive, and it is nothing to freak out over.

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[6] Ibid, pp. 9-10.

If your credences are vague, then in the language of the standard framework, we should say that there is more than one probability function in your representor. Translating into the language of supervaluationism, we should say that the expression “your credence function” is vague, and that it has many allowable sharpenings, and that these sharpenings are exactly the probability functions in your representor. Now consider any statement uses the expression “your credence function” – such as, “Option A maximizes your expected utility, calculated using *your credence function*,” – or, any other statement that is equivalent to that one – such as “Option A is rationally permissible for you.” According to supervaluatonism, such a statement is determinately true iff it comes out true no matter which one of the probability functions in your representor we plug in for your credence function. It is determinately false iff it comes out false no matter which one of those functions we plug in for your credence function. If the statement comes out true when we plug in some of these functions but false when we plug in others, it is a penumbral case, neither determinately true nor determinately false.

With these thoughts in mind, le’s return to Elga’s two-bets case. First, consider the statement that it is rationally permissible for you to take Bet A. This is equivalent to the statement that taking Bet A has a higher expected payoff than refusing it, calculated using your own credences. But if we plug in some of the allowable sharpenings of “your credence function,” this comes out true, whereas if we plug in others, it comes out false. So it is neither determinately true nor determinately false; in other words, the action of accepting Bet A is a penumbral case of rational permissibility. For similar reasons, the same thing is true of turning down Bet A, of accepting Bet B, and of turning down Bet B: All of them are borderline cases of rational acceptability. However, one thing that all of the functions in your representor agree on is that it is *not* utility-maximizing for you to turn down both bets. So if you do turn down both bets, it will be determinately true that at least one of the two acts you have lately performed was foolish – though it is indeterminate which one of those acts was the foolish one. You are determinately not-off-the-hook, as far as rationality is concerned.

Now let’s turn to Elga’s argument concerning Situation I and Situation II. Suppose that you are in Situation I: You have already turned down Bet A, and you have yet to decide about Bet B. At present, it is not definitely true that you have done anything irrational. But if you now refuse Bt B, it will then be determinately true that you have done something irrational. Does this mean that rationality requires you now to accept Bet B?

Not at all. On some of the allowable sharpenings of your credence function, your credence for H is greater than 0.6. Relative to any such sharpening, the way to maximize your expected utility is to accept Bet A and refuse Bet B. So, relative to any such sharpening, it is true that if you now refuse Bet B, you will have done something irrational – but that is only because you already made a mistake when you turned down Bet A. If you now accept Bet B, you will only be compounding your error. Therefore, relative to some of the allowable sharpenings, it is in fact rationally forbidden for you to turn down Bet B. So it is not determinately true that you are rationally required to take Bet B -- in fact it is not even determinately true that you are not forbidden to take Bet B. All of this is equally true in Situation II (as is shown on the back side of your handout). So, contrary to Elga's conclusion, it is determinately true that rationality requires exactly the same things of you in both Situation I and in Situation II – though it is indeterminate just what it does require of you in those situations.

Thus, if we interpret imprecise credences with vague credences, we can evade Elga's argument. One cost is that we must recognize that there are cases in which it is indeterminate what rationality requires of us. Such is life.