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Laws and meta-laws of nature:
Conservation laws and symmetries

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Abstract

Symmetry principles are commonly said to explain conservation laws—and were so employed even by Lagrange and Hamilton, long before Noether's theorem. But within a Hamiltonian framework, the conservation laws likewise entail the symmetries. Why, then, are symmetries explanatorily prior to conservation laws? I explain how the relation between ordinary (i.e., first-order) laws and the facts they govern (a relation involving counterfactuals) may be reproduced one level higher: as a relation between symmetries and the ordinary laws they govern. In that event, symmetries are *meta-laws*; they are not mere *byproducts* of the dynamical and force laws. Symmetries then explain conservation laws whereas conservation laws lack the modal status to explain symmetries. I elaborate the variety of natural necessity that meta-laws would possess. Proposed metaphysical accounts of natural law should aim to accommodate the distinction between meta-laws and mere byproducts of the laws just as they must accommodate the distinction between laws and accidents.

1. Introduction

It is widely known that within a Lagrangian framework, each of the classical spacetime symmetry principles logically entails one of the familiar conservation laws. Indeed, it is commonly asserted that the symmetry principles explain why the conservation laws hold.

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This purported explanation has remained largely unexamined by philosophers. One reason for finding it puzzling is that the derivation of conservation laws from symmetry principles can be run in reverse: given a fundamental dynamical law like Hamilton's principle, each of the great conservation laws entails one of the spacetime symmetry principles. What, then, makes the symmetry principles explanatorily prior to the conservation laws? Perhaps explanatory priority goes in the opposite direction: the conservation laws explain why the symmetries hold. Or perhaps neither explains the other. Rather, the fundamental laws of motion are explanatorily prior to both; they explain why the laws exhibit certain symmetries and why certain conservation laws hold.

In this paper, I specify the conditions under which symmetries have explanatory priority over conservation laws. Under these conditions, a symmetry principle constitutes a *requirement* on other laws rather than a *byproduct* of them; symmetry principles possess a stronger variety of natural necessity than conservation laws do. In that event, I argue, symmetry principles explain why conservation laws hold because symmetry principles are laws *of laws*—meta-laws governing (and helping to explain) ordinary (i.e., non-meta, "first-order") laws in a manner precisely analogous to the way in which those laws govern (and help to explain) ordinary facts and events.

I will elaborate this analogy, inspired by remarks like Wigner's characterization of a symmetry principle as "a superprinciple which is in a similar relation to the laws of nature as these are to the events" (1972, p. 10) and "as laws which the laws of nature have to obey" (1985, p. 700). If a given symmetry principle is a meta-law, then the first-order laws not only do as a matter of fact exhibit this symmetry, but also *must* exhibit it, just as in consideration of the law that all copper objects are electrically conductive, the regularity that all copper objects are electrically conductive not only does obtain, but also *has* to obtain.[1] Just as the first-order laws are able to explain various facts by rendering them necessary, so a given symmetry principle (if it is a requirement rather than a byproduct) is able to explain the corresponding conservation law.

To characterize what it would be for symmetry principles to be laws governing other laws, we must first understand what it is for ordinary, non-meta laws to govern. I shall draw upon my previous work (Lange, 1999, 2000, 2002a, 2004, 2005, 2006) on the way in which laws are set apart from accidents by virtue of their special relation to counterfactual conditionals. On this view, the laws collectively possess as much resilience under counterfactual suppositions as they logically possibly could, and such "maximal resilience" is associated with a variety of necessity. The laws's explanatory power derives from their necessity; the laws explain why *p* obtains by making *p* inevitable, unavoidable—necessary. I shall take this account and apply it one level higher in order to understand what it would be for symmetry principles to be meta-laws and hence to possess a species of necessity stronger than conservation laws possess, making symmetry principles explanatorily prior to conservation laws.[2]

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[1]Van Fraassen (1989) says, "Symmetries of the model … are 'deeper' because they tell us something beforehand about what the laws of coexistence and succession can look like" (p. 223). But equally, the laws of succession and coexistence tell us something about the symmetries of the family of models. Van Fraassen's remark that symmetries come "before" first-order laws, specifying something about what those laws *can* be like, sounds more appropriate (and accounts for the symmetries's greater "depth") if symmetries are not merely ways of describing those laws, but modally more exalted *requirements* that those laws must satisfy.

[2]That a given symmetry principle is *explanatorily* prior to the laws it governs does not require it to be prior to them *in the order of knowing*.

I begin, in Section 2, by describing what a symmetry principle is and how the spacetime symmetry principles entail the great conservation laws. I shall then explain that this derivation can be run in reverse and that Noether's theorem (often invoked in this connection) fails to reveal why the symmetry principles are explanatorily prior to the conservation laws.

In Section 3, I distinguish a symmetry principle that is a requirement on the laws (a meta-law) from a symmetry principle that is a byproduct of the laws. Symmetry under arbitrary time–displacement, for example, is a byproduct if it just happens that all of the fundamental laws of nature are time-translation invariant equations, though there is no higher-level requirement compelling them to be so. A symmetry principle that is a byproduct of the laws has no power to explain its associated conservation law. It is analogous to an accident, such as the fact that all gold cubes are smaller than one cubic mile; this regularity is a byproduct of the sizes that the actual gold cubes happen to have, and so it has no power to explain why any particular gold cube is smaller than one cubic mile. The mere statement of a symmetry principle—as a regularity among laws—fails to reveal whether that principle is purported to be a meta-law or a byproduct of the laws, just as a bare statement of an ordinary regularity (e.g., "All cubes of gold [or uranium-235] are smaller than a cubic mile") fails to specify whether that regularity is supposed to be a law or accidental.[3]

But there is a disanalogy here: even if a symmetry principle is a byproduct of the laws rather than a meta-law, it is not accidental in the manner of the gold-cubes regularity. For a symmetry principle's truth is a consequence of the laws of nature alone. A first-order regularity's being a law of nature is equivalent to its being non-accidental,[4] but a symmetry principle's being a meta-law is not equivalent to its being non-accidental, since even symmetry principles that are not meta-laws are non-accidental.

Thus, we have some work to do in order to understand what it would be for a symmetry principle to be a meta-law. One might have thought that a meta-law was simply a law that is about other laws—that it differs from first-order laws only in its content, not in its modal status. However, this is not so: the first-order laws are simply the non-accidents, but all symmetry principles are non-accidental (whether holding as meta-laws or byproducts). A meta-law is distinguished from a byproduct of the first-order laws by virtue of possessing a stronger variety of natural necessity than first-order laws possess. That is ultimately why meta-laws are able to explain first-order laws.

In Section 4, I begin working towards an account of the meta-laws's modal status by elaborating the modal status of first-order laws. They are distinguished from accidents by their stability under counterfactual suppositions. Such stability is associated with a species of natural necessity. In Section 5, I show how this distinction between first-order laws and

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[3]Occasionally, scientists state explicitly whether they regard a given symmetry principle as a meta-law or a byproduct of the laws. Consider Houtappel, Van Dam, & Wigner (1965): "[W]e regard invariance transformations as superlaws which we expect to hold not only for those laws of nature which we have come to understand but also for all others" (p. 600).

[4]One might object that not all contingent non-accidents are natural laws because some contingent logical consequences of laws are not laws. For example, if it is a law that all emeralds are green and a law that all rubies are red, then although it is no accident that all things that are emeralds or rubies are green or red, is it a law? (The reason why a certain stone is green or red is because it is a ruby, not because it is a ruby or an emerald.) However, for the sake of simplicity, I shall use "natural law" in a broad sense that includes the laws's logical consequences; my concern is the distinction between laws and accidents, not between laws and naturally necessary non-laws.

accidents is mirrored one level higher by the way that symmetries as meta-laws are set apart from symmetries as byproducts. The range of counterfactual suppositions under which the meta-laws would still have held, in connection with possessing their characteristic variety of necessity, is broader than the range of counterfactual suppositions under which the conservation laws would still have held, in connection with possessing their characteristic variety of necessity. Consequently, meta-laws possess a stronger variety of natural necessity, and so meta-laws can explain why the corresponding conservation laws hold, but not the reverse.

I am not trying to argue that there actually are meta-laws; that is for science to investigate. My aim is to understand what difference it would make whether a symmetry principle is a meta-law or a byproduct of the laws—especially what difference it would make to the symmetry principle's explanatory power. The difference between a regularity among the laws that merely obtains and one that obtains as a meta-law is a difference for which any metaphysical analysis of natural law should account—just as it should account for the difference between first-order laws and accidents. But although symmetry principles are widely considered to be among the most important results of physics, meta-laws have gone largely untreated by the leading philosophical theories of natural law. Apart from making some brief concluding remarks (in Section 6), I do not try to settle here which proposed philosophical analysis of natural law does best at accounting for meta-laws.

2. Why do symmetries explain conservation laws?

Typically, it is held that the classical conservation laws are explained by spacetime symmetries in the laws of physics:

> There are some [quantities] whose constancy is of profound significance, deriving from the fundamental homogeneity and isotropy of space and time … Let us consider first the conservation law resulting from the homogeneity of time. (Landau & Lifshitz, 1976, p. 13)

> The classical conservation laws of mechanics originate in the symmetry of space time. (Wigner, 1954a, p. 199)

> [D]ue to the invariance of the laws of physics under spatial transformations momentum is conserved. (Gross, 1996, p. 14257)

> [W]e should like to point out the usefulness of studying the conservation laws in the light of still more fundamental principles of physics. As an example, the law of conservation of momentum can be derived from the more basic concept that physical phenomena do not depend on the place where measurements are made. Such reductions of conservation laws to deeper principles may well lead to important clarifications … . (Feinberg & Goldhaber, 1963, p. 45)

However, there are notable dissenters. Brown and Holland, for instance, say that the view that a conservation law is explained by a symmetry principle

> is wrong…. The very notion of explanation here is misguided. … [There exists] a *correlation* between certain dynamical symmetries and certain conservation principles. Neither of these two kinds of thing is conceptually more fundamental

than, or used to explain the existence of, the other … . (Brown & Holland, 2004, pp. 1137–1138)

To begin sorting this out, let's get clear on what a symmetry principle says. A natural law is "symmetric" in a certain respect exactly when it remains unchanged under a certain transformation. For instance, consider Coulomb's law:

> For any two point bodies long at r_1 and r_2 with charges q_1 and q_2, body #2 exerts on body #1 an electric force $F = q_1 q_2 / |r_1 - r_2|^2$ away from #2.

Coulomb's law is unchanged under the transformation $q_1 \rightarrow -q_1$, $q_2 \rightarrow -q_2$:

$$F = q_1 q_2 / |r_1 - r_2|^2 \rightarrow F = (-q_1)(-q_2)/|r_1 - r_2|^2 = q_1 q_2 / |r_1 - r_2|^2.$$

For any system of charged bodies, the charges's signs are irrelevant to Coulomb's law; if each body's charge had been opposite in sign (but unchanged in magnitude), the forces demanded by Coulomb's law would have been no different.

Coulomb's law is also symmetric under arbitrary spatial displacement, i.e., a shift of all bodies's positions at all times by the same vector a. (Transformed by $r_i \rightarrow r_i + a$, Coulomb's law yields $F = q_1 q_2 / |[r_1 + a] - [r_2 + a]|^2 = q_1 q_2 / |r_1 - r_2|^2$.) The bodies's absolute positions are irrelevant to the law; only their separation matters.

There might have been a law privileging a given point in space. For example, it might have been a law that each body at any non-zero distance r from the universe's center feels a force q^2/r^2 toward the center. This law is not symmetric under arbitrary spatial displacement. The result of taking a possible world that (non-vacuously) accords with this law and shifting all of its bodies by a, but leaving their charges and the forces they feel (along with the location of the universe's center) unchanged, is a world that does not accord with this law.

Generalizing from one symmetry exhibited by one law, a "symmetry principle" ascribes some symmetry to the laws as a whole. We must be careful here since every logical consequence of laws is a law. Hence, if Coulomb's law is a law, then for a given moment T in the universe's history, it is a law that for any two point bodies at any time $t > T$, $F = q_1 q_2 / |r_1 - r_2|^2$. This law, unlike Coulomb's law, is not time–displacement symmetric (i.e., invariant under the transformation $t \rightarrow t \pm a$ for arbitrary temporal interval a). A world can satisfy this law while departing from Coulomb's law before T, but by shifting all events in such a world by some interval a into the future, an exception to Coulomb's law may get shifted to a time after T, thereby violating the above consequence of Coulomb's law.

However, the principle of time–displacement symmetry should not preclude this law—on pain of precluding any world governed by Coulomb's law. Accordingly, the principle should require merely that every law *follow* from time–displacement symmetric laws—i.e., that the laws *as a whole* be unchanged under time–displacement.[5] My earlier example with a law privileging the universe's center suggests that symmetry principles are not logically, conceptually, or metaphysically necessary.[6]

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[5] If we had some independent way to distinguish fundamental from derivative laws, then we could express the principle as "All *fundamental* laws are time–displacement symmetric." (Alternatively, perhaps fundamental laws are distinguished by symmetry principles and other "meta-laws".)

[6] Presuming that the natural laws are not metaphysically necessary. Scientific essentialists (such as Bigelow, Ellis, & Lierse, 1992; Ellis, 2001) reject this.

As is widely known, each of the 10 great conservation laws is connected in classical physics to a spatiotemporal symmetry principle:

symmetry:		conservation law:
time displacement	⟵⟶	energy
spatial displacement	⟵⟶	linear momentum
(in three independent directions)		(in three independent directions)
rotational	⟵⟶	angular momentum
(about three independent axes)		(about three independent axes)
under velocity boosts	⟵⟶	velocity of the center of mass
(in three independent directions)		(in three independent directions)

These symmetries (which capture the irrelevance of absolute times, positions, directions, and steady straight-line velocities), conservation laws, and connections also hold in special relativity (though with relativistic expressions for energy and momentum, and with velocity-boost symmetry involving Lorentz rather than Galilean transformations).[7]

Let us now briefly rehearse how (on the commonly held view) a given symmetry principle *explains* the associated conservation law. The explanation presupposes that for any type of force, there is a scalar function of position (a "potential" V), determined by the bodies's position coordinates and time, such that the force F of this type on a body with unit "charge" (of the kind relevant to this type of force) at a given location is in the direction from that location in which the potential diminishes most sharply and has a magnitude equal to the potential's slope in that direction. (In other words, $F = -\text{grad } V$.)[8] That is, the explanation presupposes that the work needed to put the system into a given configuration (from an arbitrarily selected starting configuration) is independent of the path through state space to that configuration (from the starting configuration). In other words, the system must have a well-defined potential energy. (The body's contribution to the system's potential energy is V times the body's charge.) Under these conditions, an isolated system (which could be the entire universe) of point bodies 1 through N (the momentary state of which consists of the bodies's coordinates $q_1, q_2, ..., q_{3N}$ and their instantaneous rates of change $q_i' = dq_i/dt$) has a well-defined Lagrangian $L(q_1, q_2, ..., q_{3N}, q_1', ..., q_{3N}', t)$ involving the difference between the system's kinetic and potential energies.

Hamilton's principle (presumed to be the fundamental law governing the system's behavior) says that the system's actual path through state space from its state at time t_1 to

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[7]Some forces (e.g., friction) disobey energy conservation. (They are covered by classical physics, obeying $F = ma$.) But in classical (and relativistic) physics, all of the *fundamental* forces are supposed to obey these conservation laws. I confine myself throughout to fundamental interactions. One might have wondered: Isn't mass conservation a "great" conservation law? No worries—it follows from momentum conservation holding in all inertial frames. Take momentum conservation holding in one inertial frame: for some constant quantity C, $\Sigma m_i v_i = C$. (A more rigorous argument would have to include the momentum carried by various fields.) Consider another inertial frame, moving with velocity V relative to the first inertial frame: $m_i \rightarrow m_i' = m_i$, $v_i \rightarrow v_i' = v_i - V$. Then the system's total momentum in the new frame is another constant $C' = \sum m_i' v_i' = \sum m_i(v_i - V) = \sum m_i v_i - V \sum m_i = C - V \sum m_i$. Hence, $\sum m_i$ is constant.

[8]Since a frictional force on a body is a function of its speed, this explanation does not cover a system with friction. Such a system can exhibit the symmetry without obeying the conservation law. For example, the equations governing a body in a viscous homogeneous liquid exhibit time–displacement and space–displacement symmetry without energy and momentum conservation. Friction, however, is not a fundamental force, and it is commonly presumed that every fundamental force can be expressed in terms of such a potential. (See note 7; Houtappel, Van Dam, & Wigner, 1965, p. 218; Havas, 1973).

its state at time t_2, as compared to nearby alternative paths between those same states at t_1 and t_2, is such that the system's $S = \int L \, dt$ is "stationary" (i.e., is a maximum, minimum, or saddle point). That is, S's variation over small variations in the path vanishes for the actual path. By the calculus of variations, S is stationary only if for each coordinate q_i, the system's path satisfies the Euler–Lagrange equation:

$$\mathrm{d}/\mathrm{d}t(\partial L/\partial q_i') - \partial L/\partial q_i = 0.$$

These $3N$ equations for $3N$ unknown functions $q_i(t)$ suffice to determine the system's path given L and the initial q_i and q_i'.

Within this framework, conservation laws are derivable from symmetry principles. Unmentioned in textbooks, however, is that these derivations can all be run in reverse. (Perhaps the reverse direction is considered obvious, but I think the reason why one direction is presented and the other omitted is that the symmetry principles are thought to explain the conservation laws, not the reverse.) Because derivations of symmetries from conservation laws are seldom seen, I shall give a simple one. Consider a system of two point bodies (masses m_1 and m_2). Let's derive the symmetry of the system's equation of motion under small spatial displacement in the x direction from the conservation of the x-component of the system's linear momentum. Where v_{1x} is the x-component of body #1's velocity and F_{1x} is the x-component of the force on body #1, Newton's second law of motion yields

$$\mathrm{d}/\mathrm{d}t(m_1 v_{1x}) = F_{1x}.$$

As presupposed earlier, F_{1x} can be expressed in terms of the system's potential energy $U(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{t})$:

$$F_{1x} = -\partial U(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{t})/\partial x_1.$$

Then body #1's equation of motion is

$$\mathrm{d}/\mathrm{d}t(m_1 v_{1x}) = -\partial/\partial x_1[U(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{t})].$$

Suppose we displace the system by a small distance a along the x-axis:

$$x_1 \to X_1 = x_1 + a,$$

$$x_2 \to X_2 = x_2 + a,$$

$$t \to T = t,$$

$$m_1 \to M_1 = m_1,$$

$$v_{1x} \to V_{1x} = v_{1x}.$$

Then body #1's equation of motion is invariant under this transformation if

$$\mathrm{d}/\mathrm{d}T(M_1 V_{1x}) = -\partial/\partial X_1[U(\boldsymbol{r}_1 + \boldsymbol{a}, \boldsymbol{r}_2 + \boldsymbol{a}, T)].$$

By the transformations, this holds exactly when

$$\mathrm{d}/\mathrm{d}t(m_1 v_{1x}) = -\partial/\partial x_1[U(\boldsymbol{r}_1 + \boldsymbol{a}, \boldsymbol{r}_2 + \boldsymbol{a}, t)].$$

For small $\boldsymbol{a}$, we can use the Taylor expansion

$$U(\boldsymbol{r}_1 + \boldsymbol{a}, \boldsymbol{r}_2 + \boldsymbol{a}, t) = U(\boldsymbol{r}_1, \boldsymbol{r}_2, t) + a[\partial U(\boldsymbol{r}_1, \boldsymbol{r}_2, t)/\partial x_1 + \partial U(\boldsymbol{r}_1, \boldsymbol{r}_2, t)/\partial x_2].$$

So body #1's equation of motion is invariant under this transformation if

$$\mathrm{d}/\mathrm{d}t(m_1 v_{1x}) = -\partial/\partial x_1[U(\boldsymbol{r}_1, \boldsymbol{r}_2, t)] - a\partial/\partial x_1[\partial U(\boldsymbol{r}_1, \boldsymbol{r}_2, t)/\partial x_1 + \partial U(\boldsymbol{r}_1, \boldsymbol{r}_2, t)/\partial x_2].$$

This holds (considering the original equation of motion) if

$$\partial U(\boldsymbol{r}_1, \boldsymbol{r}_2, t)/\partial x_1 + \partial U(\boldsymbol{r}_1, \boldsymbol{r}_2, t)/\partial x_2 = 0,$$

i.e., if

$$\mathrm{d}/\mathrm{d}t(m_1 v_{1x}) + \mathrm{d}/\mathrm{d}t(m_2 v_{2x}) = 0.$$

Hence, body #1's equation of motion is symmetric under small spatial displacement in the x direction if

$$\mathrm{d}/\mathrm{d}t(m_1 v_{1x} + m_2 v_{2x}) = 0,$$

i.e., if the x-component of the system's total linear momentum is conserved.[9]

Within the Hamiltonian framework, the spacetime symmetry principle entails the associated conservation law, but the conservation law also entails the symmetry principle. Why, then, is the symmetry principle explanatorily prior to the conservation law?

One popular way to understand the direction of explanatory priority is to invoke Noether's theorem:

> [M]omentum conservation really follows from Newton's third law of motion. But where does Newton's third law come from? Noether's theorem is the deeper statement, implying that the total momentum is conserved, *because* the interactions are determined by laws that don't depend upon where the system is located in space! (Lederman & Hill, 2004, p. 105, emphasis in the original)
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> Conservation of energy and momentum had been known for centuries … In light of Emmy Noether's insight, it is instructive to ask what symmetries are responsible. (Zee, 1986, pp. 119–121)
>
> Noether's theorem tells us … that the conservation of overall angular momentum is due to the invariance of physical laws under rotations … . (Schumm, 2004, p. 206)

The proper interpretation of Noether's theorem concerning continuous global symmetries[10] is not entirely straightforward (Brading & Brown, 2003; Brown & Holland, 2004; Havas, 1973). But my point is that none of the textbook arguments deriving conservation laws from symmetry principles appeals to Noether's theorem. The theorem generalizes these derivations but does nothing to supply them with explanatory significance.

It might be suggested that in bringing all of the individual derivations under one unifying form, Noether's theorem provides these derivations with their explanatory import. However, as I will explain in a moment, the explanatory power of these various derivations was widely appreciated long before Noether's theorem revealed these derivations to be unified, suggesting that their explanatory power does not depend on their connection through Noether's theorem. Furthermore, insofar as the derivations from symmetry principles to conservation laws can be placed within a common framework, so

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[9]Conversely, if U is spatial-displacement invariant, then $U(\boldsymbol{r}_1, \boldsymbol{r}_2, t) = U(\boldsymbol{r}_1 + \boldsymbol{a}, \boldsymbol{r}_2 + \boldsymbol{a}, t) = U(\boldsymbol{r}_1, \boldsymbol{r}_2, t) + a[\partial U(\boldsymbol{r}_1, \boldsymbol{r}_2, t)/\partial x_1 + \partial U(\boldsymbol{r}_1, \boldsymbol{r}_2, t)/\partial x_2]$, so $0 = \partial U(\boldsymbol{r}_1, \boldsymbol{r}_2, t)/\partial x_1 + \partial U(\boldsymbol{r}_1, \boldsymbol{r}_2, t)/\partial x_2 = \mathrm{d}/\mathrm{d}t(m_1 v_{1x} + m_2 v_{2x})$.

[10]That is, Noether's "first" theorem. Her "second" theorem, proved in the same 1918 paper, encompasses symmetries that are local, i.e., where the transformations under which the Lagrangian is invariant (perhaps up to a total divergence, which makes no difference to the equations of motion derived from Hamilton's Principle) may depend on arbitrary functions of spacetime.

too can the various derivations from conservation laws to symmetry principles. As Brown and Holland (2004) remark, Noether "stressed that her first 1918 theorem can be proved in reverse" (pp. 1137–1138). (See also Butterfield 2006.) The link that Noether's theorem captures between symmetries and conservation laws is (ahem!) symmetric and so cannot account for the direction of explanatory priority.

That Noether's theorem is irrelevant to a spacetime symmetry principle's power to explain the associated conservation law is strongly suggested by the fact that these explanations were given long before anything resembling Noether's theorem had been even remotely stated. For example, Hamilton derived linear and angular momentum conservation from what he announced as the fundamental principle of dynamics. Hamilton intended these derivations to be explanations of the conservation laws; Hamilton was arguing that his is the fundamental dynamical principle, and his argument was that it is responsible for (among other things) the momentum conservation laws. Besides the fundamental principle, the other premise in Hamilton's derivation is a symmetry principle:

> For this purpose, it is only necessary to observe that it evidently follows from the conception of our characteristic function V [which figures in the putative fundamental principle], that the function depends on the initial and final positions of the attracting or repelling points of a system, not as referred to any foreign standard, but only as compared with one another; and therefore that this function will not vary, if without making any real change in either initial or final configuration, or in the relation of these to each other, we alter at once all the initial and all the final positions of the points of the system, by any common motion, whether of translation or of rotation. (Hamilton 1834/1940, p. 112)

Even earlier, Lagrange emphasized that the conservation laws are explained by *his* basic principle of dynamics:

> One of the advantages of the formula under discussion is that it provides immediately the general equations which contain the principles or theorems known under the names of the Conservation des Forces Vives, conservation of the motion of the center of gravity, conservation of the motion of rotation or principle of areas … . These principles must be viewed as general results of the laws of dynamics rather than fundamental principles of this science. (Lagrange 1811/1997, p. 180)

Lagrange's explanations appeal to spacetime symmetry principles. For instance, the derivation of energy conservation employs the principle that "the functions T, V, L, M, etc. do not contain the finite variable t" (p. 233, cf. p. 212). Lagrange's explanation of why linear momentum is conserved begins:

> Let us consider a system of bodies having an arbitrary configuration and connected in any manner but without any fixity or obstacle hindering their motion. It is evident in this case that the constraints of the system can only depend on the relative positions of the bodies. Consequently, the equations of condition can only contain functions of the coordinates which define the relative distances between the bodies. (Lagrange, 1811/1997, p. 190)

Considering that the tradition of such explanations extends to the dawn of analytical mechanics, we should expect nothing like Noether's theorem to be needed to ground the

explanatory priority of symmetry principles over conservation laws. It is incorrect to appeal to Noether's theorem to secure these explanations, and it is equally incorrect to argue that since Noether's principle fails to supply a spacetime symmetry principle with the power to explain the associated conservation law, nothing does so. Regarding the view that a conservation law is explained by a symmetry principle, Brown and Holland (2004) say that it "is wrong, and not borne out by the nature of Noether's theorem" (pp. 1137–1138). But we should never have expected Noether's theorem to bear it out, so its failure to do so should not count as evidence that there is no such explanatory priority.

It might be suggested that the reason why the laws fail to privilege any absolute positions, times, spatial directions, etc. is because there are no such things (just as the reason why the laws fail to make the force between two bodies depend on how near they are to a ghost is because there are no ghosts). On this view, spacetime symmetry principles are grounded directly in a fundamental feature of the universe and so presumably they are explanatorily prior to conservation laws.[11] However, as we have just seen, even natural philosophers (such as Lagrange and Hamilton) who did not deny the reality of absolute space, time, and motion regarded symmetries as explanatorily prior to conservation laws. Even if there were absolute positions, times, directions, etc., the laws could nevertheless fail to privilege any. Under those circumstances, the symmetry principles could still explain the conservation laws. A spacetime symmetry principle's explanatory priority over the associated conservation law is presumably not dependent upon the transformation changing only features that do not exist, but are merely introduced by us in the course of describing nature.[12]

3. Requirements versus byproducts

A conservation law is associated with a regularity: that in every isolated system, a given quantity is conserved. There are two ways in which this regularity can be understood as holding as a matter of law. One way is for it to hold solely in virtue of being a logical consequence of

 (i) the fundamental dynamical law (e.g., Newton's second law of motion, Hamilton's principle),
(ii) various force laws—electric, gravitational, etc. (or, equivalently, laws specifying various potential functions), and
(iii) a "closure law": that there are no forces besides those in (ii).

The fundamental dynamical law of classical physics permits forces under which energy, momentum, etc. are not conserved. That there are no such forces (at least at the

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[11]The absence of absolute positions cannot explain why no laws refer to them, since laws can refer to uninstantiated kinds of things. (For instance, suppose a fundamental short-range force can be felt only when particles of certain kinds interact, but such particles are never near enough to interact.) To explain a conservation law, the absence of absolute positions would have to be a law, not accidental. We would then have traded our original question (Why are symmetry principles explanatorily prior to conservation laws?) for a new question: Why is the law prohibiting absolute positions explanatorily prior to conservation laws?

[12]In contrast, the laws's invariance under gauge transformations may be explained by their transforming only features that by law (note 11) fail to refer (Brading & Castellani, 2003).

fundamental level; see notes 7 and 8) is simply a consequence of the kinds of forces there actually are, according to this interpretation of the conservation law as a "byproduct" ("side-effect", "offspring") of those other laws.

As byproduct, a conservation law is a genuine law (i.e., is a natural necessity, not accidental); the regularity with which it is associated holds in every possible world governed by exactly the same force laws and fundamental dynamical law as the actual world. Nevertheless, as a byproduct, the conservation law is a kind of coincidence. If the conservation law holds as a byproduct of the laws in (i), (ii), and (iii), then there just happen to be no kinds of forces that fail to conserve energy, momentum, etc. There is no reason why, out of all of the hypothetical kinds of forces compatible with the fundamental dynamical law, the world contains only forces conserving energy, momentum, etc. The laws in (i), (ii), and (iii) explain why the conservation law holds; the conservation law explains nothing about which kinds of forces there are. Since energy conservation is merely a reflection of the particular kinds of forces there happen to be, energy might not have been conserved had there been different kinds of forces (or had the force laws governing the actual kinds of forces been different).

Rather than being a byproduct of the force laws and fundamental dynamical law, a conservation law might instead impose a restriction on the kinds of forces there could be (or have been). As Feynman says,

> When learning about the laws of physics you find that there are a large number of complicated and detailed laws, laws of gravitation, of electricity and magnetism, nuclear interactions, and so on, but across the variety of these detailed laws there sweep great general principles which all the laws seem to follow. Examples of these are the principles of conservation … All the various physical laws obey the same conservation principles. (Feynman, 1967, pp. 59, 83)

On this interpretation of conservation laws, it is no coincidence that all isolated systems conserve energy; the energy conservation law explains why interactions conserving energy are the only kinds there are. The conservation law must then be a more exalted kind of law than the various force laws—in just the way that the fundamental dynamical law is usually taken to be. For instance, to ascertain what would have happened had gravity been an inverse-cubed force, or had there been some additional kind of force alongside the actual kinds, we appeal to the fundamental dynamical law: had the force laws been different, Newton's second law of motion (let's say) would still have held. For example, Airy (1830) investigated the consequences of various weird hypothetical force laws. In wondering how bodies would have behaved under these laws, Airy held fixed Newton's second law of motion. Similarly, Ehrenfest (1917) famously showed that had gravity been an inverse-cube force or fallen off with distance at any greater rate, planetary systems would have been unstable; planets would eventually have collided with the sun or escaped from the sun's gravity. This argument also presumes that Newton's second law would still have held, had gravity been an inverse-cube force.

Likewise, if a given conservation law is a requirement that the force laws *must* satisfy, then the conservation law would still have held even if the universe had been populated by different forces. Of course, the conservation law is then still a contingent truth. It would *not* still have held had *certain* other kinds of forces been present (or if the fundamental dynamical law had been different in certain ways). But those kinds of forces are absent

from (and the actual fundamental law of dynamics still obtains in) the *closest* possible worlds with different (but otherwise unspecified) force laws.[13]

Thus, whether a conservation law is a requirement on or a byproduct of force laws makes a difference to counterfactuals and explanations. (I will have the means to make these differences precise in the following section.) In addition, how we regard evidence as confirming a given hypothetical conservation law reflects whether or not we believe that it is a byproduct of force laws if it obtains. Suppose we discover some new phenomenon and hypothesize that it involves a fundamental, exotic kind of force that has heretofore gone unrecognized and plays little role in familiar phenomena. Every force with which we are already familiar conserves energy. Does this evidence give us reason to believe that the new force conserves energy? Only if we believe that energy conservation may be a requirement imposed on the force laws. Otherwise, we regard the fact that one kind of interaction conserves energy as utterly independent from the fact that another kind does; that electrical interactions conserve energy fails to confirm that gravitational interactions do, and neither confirms that the new force does.[14] In other words, if we think that energy conservation is either a byproduct or fails to obtain generally, then if we acquire some inconclusive evidence that the new force fails to conserve energy, we do not have to balance this evidence against the fact that each of the familiar forces conserves energy, since that fact confirms nothing about whether the new force conserves energy. (Analogy: if we take it to be an accident (if it is true at all) that each of the families on my block has two children, then our circumstantial evidence that the family on the corner has no children is not mitigated by our good evidence that each of the other families on my block has two children.) The reluctance of physicists in the first decade of the twentieth century to regard radioactive emission as violating energy conservation suggests that they did not think that energy conservation holds as a byproduct if at all.[15]

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[13]Newton's third law (that for any two bodies *A* and *B*, *A*'s force on *B* is equal and opposite to *B*'s force on *A*), that any force on a body is exerted by another body, and Newton's second law together logically entail global momentum conservation. If momentum conservation is a byproduct, then Newton's third law and that any force on a body is exerted by another body is a byproduct; nothing requires that all forces be like that. The laws in (ii) and (iii) explain why Newton's third law holds and why every force on a body is exerted by another body; and (i), (ii), and (iii) explain why momentum conservation holds; but Newton's third law does not help to explain why momentum conservation holds. On the other hand, if Newton's third law, that every force on a body is exerted by another body, and Newton's second law together not only entail but also *explain why* momentum conservation holds (as Lederman & Hill (2004) seem to say, in a passage I quoted earlier), then neither Newton's third law nor that all forces on bodies are exerted by other bodies are byproducts of (ii) and (iii). Rather, they must be requirements on the forces, imposing limits on what kinds of forces there could be. In that case, momentum conservation would not be a byproduct either; it would be modally on a par with Newton's third law and that all forces on bodies are exerted by other bodies. (Of course, I have been ignoring the fact that in classical physics, Newton's third law is actually violated by electromagnetic interactions, which are retarded. The momentum conservation law that follows from symmetries includes terms for the momentum in the electromagnetic field and so holds despite Newton's third law being violated. See Lange 2002b, pp. 114–115.) Analogous remarks apply to energy conservation and (playing the role of Newton's third law and that every force on a body is exerted by another body) that all forces are central forces.

[14]Whether we think energy conservation is (if true) a byproduct or a requirement, we can justly regard energy's conservation in various examined cases as confirming its conservation in unexamined cases *involving the same forces.*

[15]Planck in 1887: "If today a quite new natural phenomenon were to be discovered, one would be able to obtain at once from [energy conservation] a law for this new effect, while otherwise there does not exist any other axiom which could be extended with the same confidence to all processes in nature" (Pais, 1986, pp. 107–108). Likewise, Feynman says that we are "confident that, because we have checked the energy conservation here, when we get a new phenomenon we can say it has to satisfy the law of conservation of energy…" (Feynman, 1967, p. 76). For more on the relation of lawhood to inductive confirmation, see Lange, 2000, pp. 111–159.

A symmetry principle can likewise be understood either as a requirement on the first-order laws or as a byproduct of them.[16] As byproduct, a symmetry principle holds solely in virtue of what the first-order laws happen to be. The invariance under a given transformation of the law governing one fundamental force has no explanation in common with the invariance of the law governing another fundamental force. That every ordinary law is invariant under the relevant transformation (or follows from ones that are) is a giant coincidence. This notion of a coincidence (albeit a naturally necessary one) seems to be what Yang (1964) has in mind when remarking that "in classical mechanics, … logically the symmetry laws were only consequences of the dynamical laws that by chance possess the symmetries" (p. 394). (Obviously, "by chance" here does not refer to an accident or some physical probability.)

As byproduct, a symmetry principle merely reflects the kinds of forces there actually are. It does not purport to describe anything about the kinds of forces there would have been, had the particulars of the force laws been different. Therefore, a byproduct symmetry principle is too weak to explain the associated conservation law understood as a requirement on the laws.

In contrast, a symmetry principle is a requirement on the first-order laws only if it would still have held even if the force laws had been different. It explains why, out of all the hypothetical force laws, there are only laws that (follow from laws that) exhibit a certain invariance. Gross (1996) is onto the contrast between byproducts and requirements when he contrasts the view of symmetries and conservation laws as mere "consequences of the dynamical laws of nature" with the view that "put[s] symmetry first, … regard[ing] the symmetry principle as the primary feature of nature that constrains the allowable dynamical laws" (p. 14256).[17] As Feynman (1967, p. 94) notes, scientists sometimes regard the invariances of known force laws as confirming that the same hypothesized symmetry principles hold of any unknown laws governing as yet undiscovered, exotic kinds of forces. This confirmation is warranted only if scientists believe that the confirmed hypothesized symmetry principles may be requirements rather than byproducts. For example, Houtappel, Van Dam, and Wigner (1965) say that spacetime symmetries are not "based on the existence of specific types of interaction" (p. 958) and

> [E]ven though we have no catalog of the possible measurements and of the laws of nature … we have reason to believe that we know the abstract group of invariances. This statement amounts to the claim that we know something about the structure of the laws of nature … even though we do not know the laws of nature themselves … . (Houtappel, Van Dam, & Wigner, 1965, p. 602) [18]

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[16]This is neither Kosso's (2000, p. 115) distinction between "fundamental" and "accidental" symmetries nor Redhead's (1975, p. 81) distinction between "universal" and "dynamical" symmetries.

[17]Gross sees this as "Einstein's great advance in 1905", though I have suggested that (e.g.) Lagrange and Hamilton had already treated symmetries as requirements.

[18]Writing about isospin symmetry, Weinberg (1992) takes "the way it would be presented today, as a fundamental fact about nuclear physics that stands on its own, independent of any detailed theory of nuclear forces" and contrasts this with the 1930's conception of such non-spacetime symmetries "as mathematical tricks; the real business of physicists was to work out the dynamical details of the forces we observe" (p. 158). This sounds like the distinction between requirements and byproducts. Plausibly, spacetime symmetries were considered requirements well before other symmetries were.

Likewise, that familiar forces are symmetric under spatial reflections was widely considered good evidence that the weak nuclear force is too, even before any phenomena involving the weak force had been examined for mirror-reflection symmetry. Accordingly, scientists were very surprised when the first experiments testing parity for weak interactions revealed parity violations (Wigner, 1984, p. 594; Gardner, 1964, pp. 239–242).[19]

Fortified with the distinction between byproducts and requirements, we can return to Brown's and Holland's remark that the conception of symmetry principles explaining conservation laws

> is wrong … . The very notion of explanation here is misguided. … Neither of these two kinds of thing is conceptually more fundamental than, or used to explain the existence of, the other … . After all, the real physics is in the Euler–Lagrange equations of motion for the fields, from which the existence of dynamical symmetries and conservation principles, if any, jointly spring. (Brown & Holland, 2004, pp. 1137–1138)

If a symmetry principle or conservation law is a byproduct, then I agree with Brown and Holland: it arises entirely from laws like the fundamental dynamical law (e.g., the Euler–Lagrange equations) and the force laws. However, a symmetry principle (or conservation law) that imposes a requirement goes further; when combined with the fundamental dynamical law, it imposes limits on the kinds of forces there *could* be. Laws like the Euler–Lagrange equations and force laws do not entail that those symmetries and conservation laws would still have held, even if there had been additional kinds of forces or the force laws for the actual kinds of forces had been different. The force laws for gravity, electromagnetism, and the other kinds of forces there actually happen to be obviously do not entail anything about what any additional forces would have been like—and so do not entail a symmetry principle or conservation law that imposes a requirement on what kinds of forces there could have been. Thus, if "the real physics" includes such a symmetry principle or conservation law, then (contrary to Brown and Holland) not all of the real physics is in laws like the fundamental dynamical law and the force laws.

Symmetry principles that are requirements add to the force laws that certain counterfactual conditionals obtain, just as p's lawhood over and above p's truth adds that p would still have held under various counterfactual circumstances. In the next two sections, I elaborate this analogy to identify precisely what it would be for symmetry principles to be requirements on the laws. In this way, I identify the reason why symmetry principles as requirements are explanatorily prior to conservation laws as requirements.

4. Laws and stability

Traditionally, laws differ from accidents by having greater persistence under counterfactual suppositions. For example, if at this moment the Stanford Linear Accelerator had been operating at full power while accelerating a material object, then all material objects would still have traveled no faster than the speed of light. But there might well have been a gold cube larger than a cubic mile had Bill Gates wanted one.

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[19]Mirror-reflection symmetry is not associated with a classical conservation law. It is not a continuous symmetry and falls outside Noether's theorem. I mention it merely to illustrate how symmetry principles are typically confirmed. For more on inductive confirmation and lawhood, see Lange, 2000, pp. 111–159.

It might be supposed that an accident has a *narrower* range of invariance under counterfactual suppositions than a law. In other words, for any law and any accident, the range of counterfactual suppositions under which the law is preserved is wider than the range of counterfactual suppositions under which the accident is preserved. However, this is not so. Suppose we have laid out on a table a large number of electric wires, all of which are made of copper. Had copper been electrically insulating, then the wires on the table would not have been much good for conducting electricity. Now look at what just happened: under the counterfactual supposition that copper is an insulator, the law that all copper objects are electrically conductive obviously fails to be preserved. But the accident that all of the wires on the table are made of copper *is* preserved. So there are counterfactual suppositions under which (in a given conversational context) a given accident is preserved but a given law is not.

However, there is an important difference between the counterfactual suppositions under which some bodies would have traveled superluminally and the counterfactual suppositions under which a gold cube would have exceeded one cubic mile. The former conditions are logically inconsistent with the laws, whereas the latter conditions are logically consistent with the laws. But this difference cannot non-circularly distinguish the laws from the accidents, since it refers to the laws. It would be circular to distinguish the laws as the truths that would still have held under every counterfactual supposition that is logically consistent with *the laws*!

I have argued (Lange, 1999, 2000, 2002a, 2004, 2005, 2006) that there is a way to avoid this circularity. Roughly speaking, the laws form a set of truths that would still have held under every counterfactual supposition that is logically consistent with the set. In contrast, the set of logical consequences of the gold-cubes accident is not preserved under every counterfactual supposition that is logically consistent with that set, since (e.g.) "Bill Gates wants a large gold cube" is so consistent.

I will now try to capture this contrast in terms of whether a given set of truths possesses a property that I call "non-nomic stability". Call a claim "non-nomic" if and only if it purports to state a fact that could be governed by laws but not concerning which facts are (or are not) laws. Non-nomic claims include "All gold cubes are smaller than a cubic mile", "All emeralds are green", "Each ^{209}Po atom has a 50% chance of decaying in the next 107 seconds", and "Every closed system conserves energy". (Some of these are laws; as I use the term "non-nomic", a non-nomic claim can nevertheless be a law.[20]) However, any of these preceded by "It is (not) a law that …" is *not* non-nomic.

Now take any non-empty set Γ of non-nomic truths containing every non-nomic logical consequence of its members. Define:

> Γ possesses *non-nomic stability* if and only if for each member m (and in every conversational context[21]), m would still have been true had p been the case—i.e., $(p \rightarrow m)$—for every non-nomic claim p that is logically consistent with Γ.

Consider the accident g: whenever the gas pedal of a certain car is depressed by x inches and the car is on a dry, flat road, then the car's acceleration is $f(x)$. Had the gas pedal on a

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[20]A law-*statement*, that is. I trust that context makes it clear whether by a "law", I mean some fact or some claim expressing it.

[21]Recall that the truth-values of counterfactual conditionals are notoriously context-sensitive. In (Lange, 2000, pp. 58–82), I examine whether there are any eccentric contexts that preclude the laws's non-nomic stability.

certain occasion been pressed a bit further, g would still have held. However, a set containing g is not non-nomically stable unless it also includes a description of the car's engine, since had the engine contained six cylinders instead of four, $\sim g$ might have held. But now to be non-nomically stable, the set must also include a description of the engine factory (since had it been different, the engine might have been different). For that matter, is g preserved (in all contexts) under "Had I been wearing an orange shirt or $\sim g$"? I think not. Therefore, to be non-nomically stable while including g, a set must include the fact that I am not wearing an orange shirt! I conclude that the only non-nomically stable set containing g is the set containing *all* non-nomic truths (which *trivially* possesses non-nomic stability, since *no* non-nomic counterfactual suppositions are logically consistent with it).[22]

Therefore, I suggest that no non-maximal set of non-nomic truths containing an accident possesses non-nomic stability, but non-nomic stability is possessed by the set containing exactly the non-nomic truths m where it is a law that m.[23] This non-nomically stable set I shall call "Λ". Of course, for some non-nomic claim m where it is a law that m (such as "Like charges repel"), "Had m not been a law, then m would still have held" may well be false. But this result fails to undermine Λ's *non-nomic* stability, since this counterfactual's antecedent is not non-nomic. So I propose that it is a law that m (where m is a non-nomic claim) exactly when m belongs to a non-maximal non-nomically stable set (i.e., a set that non-trivially possesses non-nomic stability).

Every member of a non-nomically stable set, such as Λ, would still have been true under any non-nomic counterfactual supposition logically consistent with the set (i.e., under any non-nomic counterfactual supposition under which every member of the set *could* logically possibly still have been true). That is, a non-nomically stable set is as resilient under non-nomic counterfactual perturbations as it could logically possibly be. Accordingly, a non-nomic fact m's membership in a set that non-trivially possesses non-nomic stability is associated with m's possessing some variety of *necessity*—as laws do. After all, necessity involves a kind of maximal persistence under counterfactual suppositions.

A law's necessity gives it explanatory power. That like charges *must* repel, for example, explains why in fact, every pair of like charges does. It is no accident or coincidence that in every such case, there is a repulsive force; it does not reflect some special condition that just happened to prevail each time. The reason for this regularity p is that p is required by law;

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[22]My argument that the set of *all* non-nomic truths is non-nomically stable presumes Centering: that if p and q are true, then the subjunctive conditional $p \rightarrow q$ is trivially true. Although Stalnaker–Lewis semantics presumes Centering, I believe that Centering fails in a universe where there are non-extremal objective chances. In that case, even the set of all non-nomic truths lacks non-nomic stability. However, I shall set this complication aside.

[23]Actually, I allow that a set of non-nomic truths containing some but not all of the accidents might conceivably qualify as non-nomically stable—but as a fluke, in that it would not still have been non-nomically stable had p been the case, for some p that is logically consistent with the set. So to be on the safe side, I elsewhere (2000, 2005) add nested counterfactuals to the definition of "non-nomic stability": Γ's non-nomic stability requires not just $(p \rightarrow m)$, but also $q \rightarrow (p \rightarrow m)$, $r \rightarrow (q \rightarrow (p \rightarrow m))$,... for any non-nomic claims p, q, r, ... where $\Gamma \cup \{p\}$ is logically consistent, $\Gamma \cup \{q\}$ is logically consistent, ... A happy consequence of this addition is that if Γ is non-nomically stable, then had non-nomic p been the case (where $\Gamma \cup \{p\}$ is logically consistent), then Γ would still have been non-nomically stable. That is because the nested counterfactuals $p \rightarrow (q \rightarrow m)$, $p \rightarrow (q \rightarrow (r \rightarrow m))$ that secure Γ's non-nomic stability demand that had p obtained, then the counterfactuals $(q \rightarrow m)$, $(q \rightarrow (r \rightarrow m))$, ... would have held, which secures that Γ would still have been non-nomically stable. So if it is a law that m (where m is non-nomic), then had any such p obtained, m would still have been not only *true*, but also a *law*—presuming that m is a law iff m belongs to a set that non-trivially possesses non-nomic stability. We thus account for the fact that had the Earth been much nearer to the Sun, then the Earth's climate would have been quite different but the actual natural laws would still have been laws (which is why the Earth's climate would have been so different). (See notes 26 and 28).

even if charge pairs had existed under different conditions, it would still have been the case that p. By entailing that p was unavoidable (that p would still have obtained, under every naturally possible circumstance), p's natural necessity explains why p obtains.[24]

Besides Λ, are there other non-trivially non-nomically stable sets? The non-nomic logical truths form a non-nomically stable set, and they possess a variety of necessity stronger than the natural laws's.[25] I now prove (by *reductio*) that for any two non-nomically stable sets, one must be a proper subset of the other:

> Suppose that Γ and Σ are non-nomically stable, t is a member of Γ but not of Σ, and s is a member of Σ but not of Γ.
>
> Then ($\sim s$ or $\sim t$) is logically consistent with Γ.
>
> Since Γ is non-nomically stable, every member of Γ would still have been true, had ($\sim s$ or $\sim t$) been the case.
>
> In particular, t would still have been true, had ($\sim s$ or $\sim t$) been the case. I.e., ($\sim s$ or $\sim t$)$\rightarrow t$.
>
> So t and ($\sim s$ or $\sim t$) would have held, had ($\sim s$ or $\sim t$). Hence, $\sim s$ would have held had ($\sim s$ or $\sim t$).
>
> Apply similar reasoning to Σ. Since ($\sim s$ or $\sim t$) is logically consistent with Σ and Σ is non-nomically stable, every member of Σ would still have been true had ($\sim s$ or $\sim t$) been the case.
>
> In particular, s would still have been true, had ($\sim s$ or $\sim t$) been the case. I.e., ($\sim s$ or $\sim t$)$\rightarrow s$.
>
> We have now reached an impossible conclusion: ($\sim s$ or $\sim t$)$\rightarrow$(s & $\sim s$)!

Therefore, for any two non-nomically stable sets, one must be a proper subset of the other. Since Λ is non-nomically stable but none of Λ's supersets is (except for the set of all non-nomic truths), any non-maximal non-nomically stable set must be a subset of Λ.

Many of Λ's proper subsets lack non-nomic stability. Consider the set containing exactly the non-nomic logical consequences of Coulomb's law restricted to times $t > T$. It lacks non-nomic stability: the restricted Coulomb's law would not still have held had Coulomb's law been violated at some time $t < T$. However, *some* of Λ's proper subsets *are* plausibly non-nomically stable. Perhaps the fundamental dynamical law belongs to a non-nomically stable set that omits the force laws, since (according to Airy and Ehrenfest) the dynamical law would still have held even if the force laws had been different. The dynamical law then possesses a stronger variety of natural necessity than force laws do.

This apparatus reveals precisely how a conservation law as a byproduct differs from a conservation law as a requirement. Requirements (but not byproducts) join the fundamental dynamical law (and their non-nomic logical consequences) in forming a non-nomically stable set Φ that omits the force laws. Φ's non-nomic stability requires that the conservation laws in Φ would still have held even if the force laws had been different (e.g., even if there had been an additional fundamental force governed by some unspecified

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[24] Analogous remarks apply to other species of necessity. For example, that 23 cannot be divided evenly by 3 explains why each time mother tries to divide 23 strawberries equally among her three children without cutting any strawberries, she fails. (Braine, 1972, p. 144)

[25] Throughout I mean "logical truths" broadly—including mathematical and conceptual truths, for example. Some proper subsets of the set of logical truths may even be non-nomically stable. Although Λ includes all of the logical truths, for some purposes we might want to construe the "natural laws" more narrowly as the truths lacking these other varieties of necessity and belonging to a non-maximal set that possesses non-nomic stability.

law). As we saw earlier, this counterfactual holds if the conservation law imposes a requirement on the force laws but fails if the conservation law is a byproduct of the particular forces there are.

As a requirement on the force laws, a conservation law possesses a stronger variety of natural necessity than the force laws do; Φ forms a higher stratum of laws than Λ. That is, the range of counterfactual suppositions under which Φ's members would still have held in connection with Φ's non-nomic stability includes and extends beyond the range under which Λ's members would still have held in connection with Λ's non-nomic stability.[26] (But Φ's non-nomic stability does *not* require Φ's members still to have held had there been a fundamental force governed by a law that, with the dynamical law, entails these conservation laws to be violated. That counterfactual supposition is logically inconsistent with Φ and so outside the range under which Φ must be preserved in order for Φ to qualify as possessing non-nomic stability.)

This conception of conservation laws as requirements fits with Wigner (1972) remark: "[F]or those [conservation laws] which derive from the geometrical principles of invariance it is clear that their validity transcends that of any special theory—gravitational, electromagnetic, etc.—which are only loosely connected …." (p. 13) A conservation law explained by spacetime symmetries would still have held, had those "special theories" been different. It is a requirement, not a byproduct (i.e., a coincidence of the "loosely connected" force laws). Let's see why.

5. Symmetries as meta-laws

A symmetry principle differs crucially from a conservation law (whether byproduct or requirement). The regularity associated with a conservation law (that a given quantity is conserved in every isolated system) does not concern laws. That is, a statement of that regularity is a non-nomic claim; it belongs to Λ. In contrast, the regularity associated with a symmetry principle (each non-nomic truth m where it is a law that m is invariant under a certain transformation or follows from ones that are) *concerns laws*; it is a regularity in the regularities *associated with laws* governing non-nomic facts.[27] The regularity associated with a symmetry principle is thus ineligible for membership in Λ. It is "meta" to the "ordinary laws"—the first-order laws, the laws governing the non-nomic facts (i.e., laws of the form "It is a law that m" where m is non-nomic). A truth "meta" to the laws in Λ is a "meta-law" exactly when it requires something of them (rather than being their byproduct), as I shall now elaborate. Symmetry principles as requirements possess a stronger variety of natural necessity than conservation laws as requirements do, empowering the symmetry principles to explain the conservation laws and preventing the reverse.

To elaborate the notion of a meta-law, we must take the relation between laws in Λ and the non-nomic facts they govern and reproduce it one level higher. Let's begin by turning

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[26]Once we have included nested counterfactuals in the definition of "non-nomic stability" (see note 23), Φ's non-nomic stability entails that Φ would still have been non-nomically stable (and hence its members would still have been laws) had (e.g.) Coulomb's law been false. This result will come in handy in note 28.

[27]"[I]nvariance principles can be formulated only if one admits the existence of two types of information [:] initial conditions and laws of nature. It would be very difficult to find a meaning for invariance principles if the two categories of our knowledge of the physical world could no longer be sharply separated" (Houtappel, Van Dam, & Wigner, 1965, p. 596).

from the non-nomic claims to a broader set. Let a claim be "nomic" exactly when it is "non-nomic" or purports to state a fact concerning what laws govern the non-nomic facts—but not concerning which of *those* facts are (or are not) laws. That all emeralds are green is a non-nomic fact (and therefore a nomic fact); that *it is a law* that all emeralds are green is a nomic fact (and not a non-nomic fact), and likewise for the fact that *it is not a law* that all gold cubes are smaller than a cubic mile. That all laws governing non-nomic facts are time–displacement symmetric (or follow from ones that are) is a nomic fact (and not a non-nomic fact). But the nomic facts do not include the fact that *it is a law that* all laws governing non-nomic facts are time–displacement symmetric. (That's a meta-nomic fact.)

Take a non-empty set Γ of nomic truths containing every nomic logical consequence of its members. Here's the nomic analog to non-nomic stability:

> Γ possesses *nomic stability* if and only if for each member m (and in every conversational context), m would still have been true had p been the case—for every nomic claim p that is logically consistent with Γ.

Λ lacks nomic stability: If it is a law that m (where m is non-nomic), then "Had m not been a law, m would still have held" is typically false, but this counterfactual's antecedent is a nomic claim that is logically consistent with Λ, so Λ's nomic stability requires this conditional's truth. However, consider the set Λ^+ specifying which non-nomic claims are laws and which are not. I have argued (Lange, 2000) that Λ^+ possesses nomic stability. For example, Coulomb's law would still have been a law (and g would still not have stated a law) had I worn an orange shirt. Λ^+ includes statements of all regularities associated with symmetry principles (requirement or byproduct) holding of the laws governing non-nomic facts. (For example, Λ^+ includes "Each non-nomic truth m where it is a law that m is time-translation invariant or follows from ones that are".)

A non-maximal nomically stable set's members are non-trivially all invariant under every nomic counterfactual supposition logically consistent with them all collectively. The set is as collectively invariant under nomic counterfactual suppositions as it could be. Accordingly, membership in such a set is associated with possession of a variety of necessity.

Besides Λ^+ and the set of logical truths, there may be another non-maximal nomically stable set. I suggested earlier that if the laws governing non-nomic facts exhibit a certain symmetry, then the symmetry principle imposes a requirement on those laws only if it would still have held, had those laws been different. Now we can be more precise. For some symmetry principles to be meta-laws (i.e., requirements) is for them (plus their nomic logical consequences) to belong to a nomically stable set that omits various members of Λ^+ including the dynamical law, the force laws, and all of the other first-order laws. This demands that those symmetry principles would still have held had the fundamental dynamical laws been different, or had the force laws governing the actual kinds of forces been different, or had there been an additional kind of force besides the actual kinds. (But it does *not* require that they would still have held had there been an additional fundamental force such that the first-order laws *violate* the symmetry principles. That counterfactual antecedent is logically inconsistent with the set of symmetry principles and so outside the range of counterfactual suppositions under which the set must be preserved in order for it to possess nomic stability.) Some symmetry principles may be meta-laws while others hold as byproducts of the ordinary laws—"by chance", as Yang put it.

Suppose various symmetry principles to be meta-laws. The associated conservation laws do not join them to form a nomically stable set (or form a nomically stable set themselves). That's because had the fundamental dynamical law been different, the symmetries would still have held but the conservation laws need not have. For example, had the fundamental dynamical law been $F = mv$ rather than $F = ma$, the force laws would still have been temporally homogeneous and spatially isotropic, but energy and angular momentum would not have been conserved (Wigner, 1954b, pp. 437–438). A symmetry principle that imposes a requirement transcends the Hamiltonian framework. As another example, suppose it had been a law that when two bodies of unequal mass collide, the less massive one disappears and the more massive one continues moving as it would have had no collision taken place. The familiar symmetries would still have held, but not the conservation laws. (I will say more about these examples in a moment.)

Unlike even conservation laws that are requirements, required symmetries would still have held, had the fundamental dynamical law (or any of the laws governing non-nomic facts) been different. The range of counterfactual suppositions under which the meta-laws would still have held, by their nomic stability, includes and extends beyond the range under which Φ (containing dynamic laws and conservation laws that are requirements) would still have held, by its non-nomic stability. Therefore, even conservation laws that are requirements cannot help to explain the corresponding symmetry principles, since a conservation law's invariance under counterfactual suppositions is not broad enough to give the symmetry principle its requisite invariance under counterfactual suppositions. The conservation law that is a requirement cannot explain why the symmetry would still have held had the fundamental dynamical laws been different.

Notice that the counterfactual suppositions that I called upon in order to show that the conservation laws by themselves fail to form a nomically stable set are logically consistent with the conservation laws. For example, I considered the supposition s that it is a law that when two bodies of unequal mass collide, the less massive body disappears and the more massive body goes on its way as if the other body had not been there. If it is a natural law that there is only one body in the universe's entire history, that its mass remains unchanged, and that it moves uniformly forever, then s holds (vacuously) and it is also a law that energy, momentum, etc. are conserved. So the conservation laws *could* still have been laws under s. But they *would not* still have been; rather, there would still have been many bodies in the universe. So the conservation laws do not form a nomically stable set.[28]

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[28]It might be suspected that "It is a law that energy is conserved," "It is a law that momentum is conserved," etc., along with the lawhood of the fundamental dynamical law form a nomically stable set Φ^+ (that includes their logical consequences among the nomic claims, such as the symmetry principles). Indeed, this set's nomic stability might seem required in order for the conservation laws genuinely to impose requirements on the force laws, since those requirements seems to involve the truth of counterfactuals like "Had Coulomb's law not been a law of nature, then momentum conservation would still have been a law," and this counterfactual's truth is not required by Φ's non-nomic stability (since neither this counterfactual's antecedent nor its consequent is a non-nomic claim). (I am very grateful to a referee for suggesting this argument.) However, Φ^+ in fact fails to be nomically stable. For example, s is logically consistent with the conservation laws together with $F = ma$ (once again, they all hold— some vacuously—in a universe where it is a law that there is always just a single particle with constant mass moving uniformly forever), but Φ^+ is not preserved under s. As another example, consider the counterfactual supposition that it is a law that the sum of every body's $(mv)^{1/2}$ is a conserved quantity. This supposition is logically consistent with Φ^+. (In a universe where it is a law that there is nothing but a single point body of constant mass moving uniformly forever, it is a law that $\Sigma(mv)^{1/2}$ is conserved and all of the actual conservation and dynamical laws are still laws too.) But (in at least some conversational contexts, it is true that) had this

In contrast, the set of symmetry principles (if symmetries are requirements) together perhaps with other meta-laws is nomically stable. Whereas a counterfactual supposition under which the symmetry principles fail to be preserved must be logically inconsistent with the meta-laws, the conservation laws fail to be preserved even under some counterfactual suppositions with which they are together logically consistent.

Of course, just as there are some counterfactual suppositions under which the symmetry principles are preserved but the conservation laws are not, so likewise there are some counterfactual suppositions under which the conservation laws are preserved but the symmetry principles are not—for example, "Had it been a non-vacuous law that each body always moves at 5 m/s in the $+x$ direction." It is *not* the case that symmetry principles as requirements are invariant under a broader range of counterfactual suppositions than conservation laws are as requirements, just as it is not the case that a law is invariant under a broader range of counterfactual suppositions than an accident is (as I mentioned near the start of Section 4). Rather, the range of counterfactual suppositions under which symmetry principles as requirements are invariant *in virtue of which they possess a species of necessity* (namely, in connection with their nomic stability) is broader than the range of counterfactual suppositions under which conservation laws as requirements are invariant *in virtue of which they possess a species of necessity* (namely, in connection with Φ's non-nomic stability). The counterfactual supposition positing that it is a non-vacuous law that each body moves always at 5 m/s in the $+x$ direction is logically inconsistent with the set of symmetry principles (in particular, with rotational symmetry) and so falls outside the range of counterfactual suppositions under which that set must be preserved for it to qualify as nomically stable.[29] In contrast, there are (as we have seen) nomic claims that are logically consistent with the set of conservation laws but under which the conservation laws are not all preserved as laws, thereby failing to qualify as nomically stable. There is hence no

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(*footnote continued*)

supposition held, then some of the actual conservation laws would not still have held; the momentum conservation law or energy conservation law would presumably have been replaced by the law of the conservation of $\Sigma(mv)^{1/2}$. Hence, Φ^+ lacks nomic stability.

Furthermore, counterfactuals like "Had Coulomb's law not been a law of nature, then momentum conservation would still have been a law" can be accounted for solely by Φ's non-nomic stability, once "non-nomic stability" has been fortified to include nested counterfactuals (see note 23). If the strata of natural laws among the non-nomic truths are cashed out as the non-maximal non-nomically stable sets, then (as I explained in note 26) thanks to those nested counterfactuals, Φ's non-nomic stability implies "Had Coulomb's law been false, then momentum conservation would still have been *a law*." Furthermore, had Coulomb's law been false, then Coulomb's law would not have been a law (since laws, like accidents, are truths). In addition, just as there would have been a body accelerated from rest to beyond the speed of light had there been no law prohibiting such a thing, so likewise had Coulomb's law not been a law, it would not have been true. Thus we have $p \rightarrow q$ (Coulomb's not a law $\rightarrow$ Coulomb's false), $q \rightarrow r$ (Coulomb's false $\rightarrow$ momentum conservation still law), and $q \rightarrow p$ (Coulomb's false $\rightarrow$ Coulomb's not a law), from which it logically follows (by a principle of counterfactual logic; see Lewis, 1973, p. 33) that $p \rightarrow r$ (Coulomb's not a law $\rightarrow$ momentum conservation still law), which was our target counterfactual.

[29]If it is a law that there are no bodies, then vacuously each body moves always in the $+x$ direction, but rotational symmetry is not violated since the $+x$ direction is not privileged in this regard. (Recall from Section 2 that a symmetry principle pertains to the laws as a whole.) If the symmetry principles are meta-laws (i.e., requirements that the first-order laws must satisfy), then had it been a law that each body moves always in the $+x$ direction (with no further qualification precluding a law that there are no bodies), the symmetry principles would still have held, and so it would have been a law that there are no bodies. (Admittedly, it is easy to conclude hastily that this counterfactual's antecedent is logically inconsistent with the symmetry principles and hence that the symmetry principles are not preserved under it.) Likewise, it is a law that each ghost moves always in the $+x$ direction since it is a law that there are no ghosts.

"symmetry" (!) between the symmetry principles and the conservation laws, even though each is preserved under some counterfactual suppositions under which the other is not.

The reason for emphasizing not simply the range of counterfactual suppositions under which the set of symmetry principles would still have held, but instead the range under which they would still have held *by the set's nomic stability*, is that as I suggested earlier, stability is associated with a variety of necessity and it is by virtue of a law's necessity that it possesses its distinctive explanatory power: *p*'s lawhood explains why *p* obtains by rendering *p* inevitable, unavoidable—necessary. In short, I focus on stability, not on just any old range of invariance under counterfactual suppositions, because (I have argued) a set's stability involves its exhibiting *maximal* invariance (i.e., its members are together as invariant as they could together be) and so involves its members possessing a species of necessity, and necessity in turn (as we saw in Section 4) is associated with explanatory power. Symmetry principles are explanatorily prior to conservation laws because symmetry principles possess a species of necessity (associated with their nomic stability) involving invariance under a range of counterfactual suppositions that is broader than the range under which the conservation laws are invariant in connection with their species of necessity (associated with Φ's non-nomic stability).

6. Conclusion

I have argued that symmetry principles are explanatorily prior to conservation laws when symmetry principles are meta-laws governing first-order laws. I have elaborated meta-lawhood in terms of nomic stability; the relation between meta-laws and the laws they govern thus mirrors the relation between first-order laws and the facts they govern.

When a conservation law is explained by a symmetry principle, the symmetry principle functions as the "covering law" and the fundamental dynamical law functions as the "initial condition". That is, the dynamical law is governed by the symmetry principle; the symmetry would still have held even if the dynamical law had not. The symmetry principle, in belonging to a non-maximal nomically stable set, possesses a variety of necessity. It explains the conservation law by making it likewise necessary that the conservation law hold *given* the dynamical law. That is (using the corresponding variety of possibility), the conservation law holds in every possible world where the dynamical law holds. The conservation law that is explained by a symmetry principle is a requirement, not a byproduct. (Recall Wigner's remark quoted at the close of Section 4.) It joins the fundamental dynamical law in a non-nomically stable set Φ that omits the force laws. That the conservation laws would still have held, had the force laws been different (as per Φ's non-nomic stability), is ensured by the fact that not only the fundamental dynamical law, but also the symmetries would still have held, had the force laws been different (as per the symmetries's nomic stability).

It might be objected that counterlegals such as "Had the force laws been different, the symmetry principles would still have held" are too exotic to be accessible to empirical investigation. However, why does their counterlegality make them any more remote from observation than other counterfactuals? I have noted cases where scientists take evidence as confirming not merely that actual, as yet undiscovered kinds of forces obey some hypothesized conservation law or symmetry principle, but also that the kinds of forces that would have existed under various counterfactual suppositions do so. Evidence bears on the

forces there would have been for the same reason as it bears on the unknown forces there are—just as the fact that all examined emeralds are green confirms not only that the actual emeralds lying undiscovered in far-off lands are green, but also that an emerald in my pocket (had there been one) would have been green. (Which of these facts is more "remote" from our observations?) Facts about what would have been are confirmed right along with facts about what is.

Considering the scientific importance of meta-laws, it is surprising that metaphysical analyses of natural law have seldom been asked to account for them. Here I can do no more than gesture toward some of the obstacles that proposed accounts of natural law would encounter in trying to deal with meta-laws.

- The Armstrong–Dretske–Tooley account identifies laws with contingent "nomic necessitation" relations among universals. Is membership in different non-nomically stable sets (such as Λ and Φ), corresponding to different species of natural necessity, associated with different species of "nomic necessitation" relations? Does a meta-law involve a further species? For instance, does the time–displacement symmetry meta-law involve F-ness standing in a kind of nomic-necessitation relation to G-ness, where F-ness is the property of being the consequence of a (distinct kind of?) nomic-necessitation relation among universals, and G-ness is being time-translation invariant?[30]
- Lewis's account identifies the laws as the generalizations in the "best system" of truths. If Λ and Φ are associated with different varieties of natural necessity, then are Λ and Φ tied for "best system"? Would the meta-laws be the members of the best system of truths *about the first-order laws* (i.e., about the best system of truths about the Humean mosaic)?
- According to scientific essentialism, laws are metaphysically necessary: it is part of electric charge's essence, for example, that it involves the causal power to exert and to feel forces in accordance with certain particular laws. Essentialism takes counterfactuals such as "Had I worn an orange shirt, then gravity would still have declined with the square of the distance" to be grounded in essences (in this case, gravity's). The actual kinds of forces are fixed by the *world's* essence, making it true that the same kinds of forces would have existed had I worn an orange shirt (Bigelow, Ellis, & Lierse, 1992; Ellis, 2001, pp. 205, 275–276). But what essence is available to make it true that a given symmetry (or fundamental dynamical law) would still have held *had there been different kinds of forces*? According to Ellis (2001), a counterfactual holds exactly when its consequent holds "in a world of the same natural kind as ours in which the antecedent condition is satisfied, other things being as near as possible to the way they actually are" (p. 278). But by Ellis's lights, a world with different kinds of forces is not "of the same natural kind as ours".

Philosophical analyses of natural law should be asked to account not only for the distinction between first-order laws and accidents, but also for the distinction between meta-laws and byproducts of the ordinary laws.

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[30] Armstrong (1983) construes functional laws as "laws governing laws"—as involving a nomic-necessitation relation's holding among higher-order universals (i.e., properties of properties). The nomic-necessitation relation in Armstrong's meta-laws is the same as in his first-order laws. But functional laws do not involve the property of being a property standing in a certain sort of nomic-necessitation relation.

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