European Journal for Philosophy of Science (2019) 9:33
https://doi.org/10.1007/s13194-019-0256-6

PAPER IN GENERAL PHILOSOPHY OF SCIENCE

The principle of virtual work, counterfactuals, and the avoidance of physics

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Received: 7 November 2018 / Accepted: 6 May 2019/Published online: 29 May 2019

Abstract
Wilson (2017) derives various broad philosophical morals from the scientific role played by the Principle of Virtual Work (PVW). He argues roughly (i) that PVW conditionals cannot be understood in terms of things as large as possible worlds; (ii) that PVW conditionals are peculiar and so cannot be accommodated by general accounts of counterfactuals, thereby reflecting the piecemeal character of scientific practice and standing at odds with the one-size-fits-all approach of "analytic metaphysicians"; and (iii) that PVW counterfactuals are not made true partly by natural laws. I distinguish, elaborate and critically examine various arguments for these morals suggested by the PVW and Wilson's text, looking especially at what makes a displacement "virtual" and the operation of the conditionals that the PVW takes to express necessary and sufficient conditions for equilibrium. Ultimately, I do not find the PVW to be especially well suited to support Wilson's morals; some of these arguments fail, whereas others arise from general considerations rather than having to appeal to anything like the PVW.

Keywords Principle of virtual work · Statics · Mark Wilson · Counterfactual conditionals · James Woodward · Possible worlds · Laws of nature

1 Introduction

The Principle of Virtual Work (PVW) "occupies a central position in structural analysis" (Neal 1964: 1), statics, and mechanical engineering. Unfortunately, it has received relatively little attention from philosophers. A notable exception is Mark Wilson, for whom the PVW is one of the principal case studies in *Physics Avoidance: Essays in Conceptual Strategy* (Wilson 2017). Wilson regards this case study as supporting various broad morals regarding counterfactual conditionals, laws of nature,

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metaphysics, possible worlds, and the structure, scope, and strategies of classical mechanics. In this paper, I will examine the PVW and the ways that Wilson purports to derive morals from it. Ultimately, I will not find the PVW to be especially well suited to support Wilson's morals; some of the arguments suggested either by Wilson's text or, at least, by some readings of the PVW seem to me not to succeed, whereas others arise from general considerations rather than having to appeal to anything like the PVW.

In section 2, I will sketch the PVW and agree with Wilson that it exemplifies the strategy of "physics avoidance". I will explain how Wilson interprets the PVW as involving counterfactual conditionals of the sort that figure in Woodward's (2003) account of the counterfactual dependence associated with causal relations. In section 3, I will state three interrelated morals that Wilson regards the PVW as supporting. These are roughly (i) that PVW conditionals cannot be understood in terms of things as large as possible worlds; (ii) that PVW conditionals are peculiar and so cannot be accommodated by general accounts of counterfactuals, thereby reflecting the piecemeal character of scientific practice and standing at odds with the one-size-fits-all approach of "analytic metaphysicians"; and (iii) that PVW counterfactuals are not made true partly by natural laws.

After looking at several unsuccessful arguments for these morals that might be extracted from Wilson's remarks, I will turn in section 4 to what I take to be Wilson's principal motivation for these morals: that the infinitesimal displacements posited by PVW conditionals are *virtual*. I will distinguish several arguments that might be given to derive Wilson's morals from the virtual character of PVW displacements. I will argue that the displacements' virtual character either fails to support Wilson's morals or does so no differently from ordinary counterfactuals involving no virtual displacements. In section 5, I will give some reasons why we might reject a presupposition of all of these arguments for Wilson's morals: Wilson's identification of PVW conditionals with Woodwardian, manipulationist counterfactuals. The PVW might well be interpreted as involving no counterfactuals at all, but instead as stating that a given mathematical procedure's having a particular outcome when applied to a given system is a necessary and sufficient condition for the system's equilibrium. Finally, in section 6, I will explore a question about the PVW that Wilson does not much investigate: why does the PVW hold? A "bottom-up" explanation portrays the PVW as a coincidence, with different forces obeying it for different reasons. By contrast, the PVW has a "top-down" explanation iff there is a common reason why the PVW holds for every kind of force there could be. In the latter case, the PVW does not depend on the particular kinds of forces there are; a "bottom-up" derivation cannot explain it.

In the course of this paper, I will distinguish, elaborate and critically examine several arguments that Wilson may be making or, at least, that the PVW suggests. Some of these arguments for Wilson's morals, while suggested by aspects of the PVW to which Wilson attends, are (I believe) not Wilson's own arguments, whereas other arguments that I examine are more plausibly attributed to him. Although I will indicate the textual basis for attributing some of these arguments to Wilson, I am not primarily concerned with Wilson exegesis. I regard all of these arguments as worthy of careful examination. The PVW is fascinating and underdiscussed, and the topics that Wilson addresses are very important. A clearer understanding of how the PVW works and of the conditionals associated with it would be valuable in itself. In pursuing this aim, I intend to be paying proper tribute to Wilson's provocative book (and perhaps helping to make it more accessible).

2 The principle of virtual work and manipulationist counterfactuals

The "work" W performed on a body by a constant force F in the direction of the body's motion, when the body moves a distance s, is defined as the product of that force's magnitude and the distance: $W = Fs$. (If the force is opposite to the direction of motion, then $W < 0$.) If the body is constrained to move in a given direction (as when it is on a flat table top and so cannot move downward), then the work done by the force on the body is the magnitude of the force's component in the direction of the body's motion multiplied by the distance moved. This definition is usually expressed as $W = \boldsymbol{F} \cdot \boldsymbol{s}$ (with vector quantities in **bold**). Where the force changes while the body moves, the work $\mathrm{d}W$ done by $\boldsymbol{F}$ during a given infinitesimal element $\mathrm{d}\boldsymbol{s}$ of the body's motion is defined as $\boldsymbol{F} \cdot \mathrm{d}\boldsymbol{s}$; the total work is the sum of these $\mathrm{d}W$'s over the body's path: $W = \int \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{s}$.

The Principle of Virtual Work (PVW) specifies necessary and sufficient conditions for a physical system to be in equilibrium (i.e., for all of the system's components to remain where they are, as long as external conditions remain unchanged and all of the system's components are initially at rest and unchanging intrinsically).[1] Let's get a rough sense of what the PVW says, deferring certain important details until later. Consider a physical system. (It could consist of various macroscopic bodies connected by rigid rods or inextensible ropes or extensible springs, perhaps feeling Earth's gravity and other external forces applied to various parts of the system, such as by someone tugging on a rope.) The system is under various "constraints" – typically, that various components of the system will persist, remain impenetrable, remain rigid (i.e., the same size and shape), and remain in contact either at specific points (such as when two bodies must remain attached at their ends) or at some point or other (such as when a wheel must remain on a track, though it may roll). Within these constraints, the system's components are free to move, depending upon the forces on them. Suppose some bodies ($i = 1, 2, \ldots$) in the system undergo infinitesimal displacements ($\mathrm{d}\boldsymbol{s}_1$, $\mathrm{d}\boldsymbol{s}_2, \ldots$) in a manner allowed by the constraints. (These $\mathrm{d}\boldsymbol{s}_i$ are called "virtual displacements" for reasons I will later discuss but will ignore for now. In section 4, I will unpack what makes a displacement or some work "virtual" and suggest that Wilson's main arguments turn crucially on these considerations.) Consider the forces ($\boldsymbol{F}_1$, $\boldsymbol{F}_2, \ldots$) felt by these displaced bodies other than the forces imposing the constraints on them. These non-constraint forces do "virtual" work ($\boldsymbol{F}_1 \cdot \mathrm{d}\boldsymbol{s}_1$, $\boldsymbol{F}_2 \cdot \mathrm{d}\boldsymbol{s}_2, \ldots$) on the bodies as they undergo this displacement. The PVW says that the total virtual work is zero (i.e., $\sum_i \boldsymbol{F}_i \cdot \mathrm{d}\boldsymbol{s}_i = 0$) for every allowed displacement if and only if the system is in equilibrium.[2]

As a simple illustration (that Wilson (168) also uses), consider the horizontal seesaw in Fig. 1 – feeling no friction and no external forces except gravity. The constraints are the expected ones, such as that the masses (m_1 and m_2) cannot move along or fall off the lever arms, which must remain rigid and affixed to the fulcrum, which must remain at rest. The only virtual displacement allowed by the constraints is an infinitesimal change $\mathrm{d}\theta$ in the seesaw's angle (Fig. 2) – the same angle for both masses since the lever arm is rigid. Gravity pulls downward on mass m_i with a force $F_i = m_i g$. Hence (with $\sin(\mathrm{d}\theta) =$

[1] Sometimes (as in Routh 1896:142), "PVW" refers only to the principle giving necessary conditions for equilibrium; the PVW's converse then gives sufficient conditions.

[2] Oliveira (2014: 70ff.) traces the PVW from Jean Bernoulli through Lazare Carnot, Laplace, Fourier, Laplace, Lagrange, and others.

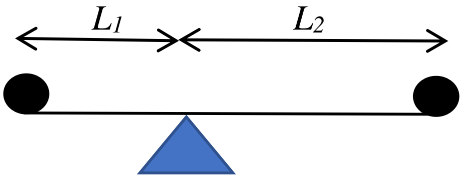

Fig. 1 A seesaw

dθ for infinitesimal dθ, so vertical $\mathrm{d}s_i = L_i\mathrm{d}\theta$), $\mathrm{d}W_1 = F_1\ \mathrm{d}s_1 = (m_1 g)\ (-L_1\mathrm{d}\theta) = -m_1 g L_1\mathrm{d}\theta$ and $\mathrm{d}W_2 = m_2 g L_2\mathrm{d}\theta$. By the PVW, the seesaw is at equilibrium iff $0 = \mathrm{d}W_1 + \mathrm{d}W_2$, i.e., iff $m_1 L_1 = m_2 L_2$, which is Archimedes' familiar law of the lever.

One of Wilson's main points is that the PVW enables us to avoid having to know the forces of constraint; to determine a system's equilibrium conditions, we do not need to know the microcomplexities of how (e.g.) the seesaw's lever-arm remains rigid or how a mass remains affixed to it (e.g., pp. 254, 306). Wilson calls this "physics avoidance." This point generalizes beyond the microcomplexities to which Wilson refers: to determine a system's equilibrium conditions, we do not need to know even the system's internal *macro*scopic mechanism, merely the displacements that the internal mechanism allows the system's freely movable components to undergo. These points have been widely appreciated (e.g., Lanczos 1949: 74; Mach 1960: 75; Planck 1933: 169), but let's illustrate them briefly. Consider the system in Fig. 3, which shows a (frictionless) pulley with an inextensible rope draped across it, the left end of which can be pulled and the right end of which goes into a mechanism hidden inside a black box, from which hangs weight *w*. If we are using Newton's laws or other bottom-up approaches, then even if we know the constraints (e.g., that the only allowable motions are the inextensible rope going down or up at the top left and the weight rising or descending), we need to know the internal mechanism in order to infer the force *P* that would produce equilibrium by pulling downward on the rope. Even given the internal mechanism (Fig. 4), it is tedious working out when *P* makes the forces all add to zero. If F_{ij} is the tension on the rope pulling pulley *i* towards pulley *j*, then the configuration of pulleys (and Newton's third law that action equals reaction) ensures that

$$F_{24} = F_{42}$$
$$F_{23} = F_{32}$$

That the system is at equilibrium ensures that the force pulling the rope clockwise around a pulley is equal to the force pulling it counterclockwise around that pulley:

$$P = F_{41}$$
$$F_{24} = F_{23}$$
$$F_{42} = F_{41}$$

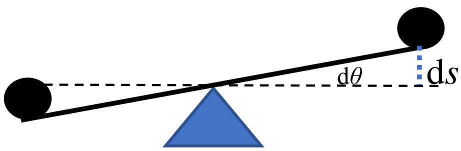

Fig. 2 A seesaw under virtual displacement

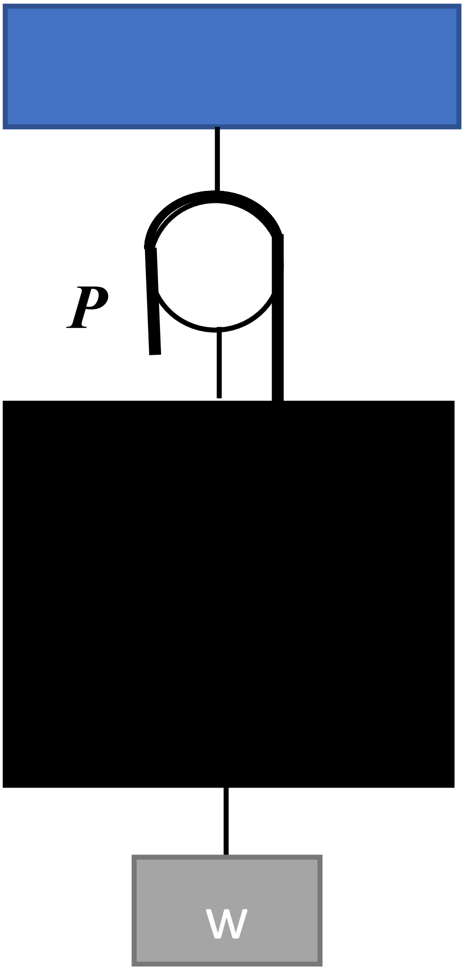

Fig. 3 A pulley, rope, and weight, with the rest of the mechanism hidden

That the downward pull on the weight is equilibrated by the upward pull on the weight is that

$$w = F_{42} + F_{41} + F_{32} + F_{32}$$

(one term on the right for each of the upward pulls on pulley 4 and thereby on the weight, with the two F_{32}'s for the pulls on pulley 4 from pulley 3 resulting from 3's two connections to pulley 2). It follows that equilibrium obtains iff $w = 4P$. By contrast, PVW avoids this mess. That a pull on the rope end of distance Δx results in the weight's rising by a distance $\Delta z = \Delta x/4$ can be inferred by simple induction from the results of some pulls – by what Wilson (316) refers to as "direct induction upon manipulative experimentation, such as the tinkerings of our do-it-yourselfer." Then we apply PVW: equilibrium holds iff $P\ \mathrm{d}x - w\ \mathrm{d}z = 0$. So since $4\ \mathrm{d}z = \mathrm{d}x$, equilibrium holds iff $w = 4P$.

Wilson interprets the PVW as specifying necessary and sufficient conditions for a system's equilibrium in terms of counterfactual conditionals: that for any virtual

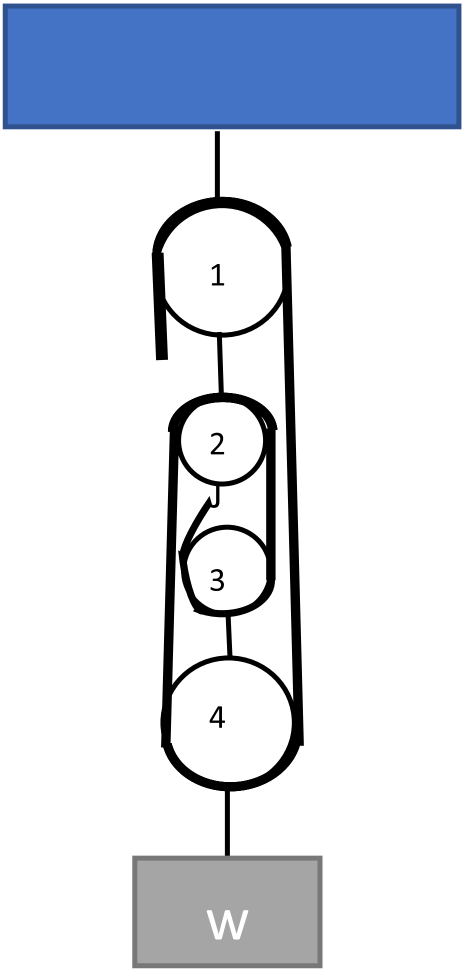

Fig. 4 The same system as in figure 3, but with the entire mechanism revealed

displacements allowed by the constraints, if the system had been so virtually displaced, then the total work thereby done on it by the non-constraint forces would have been zero. (For instance, Wilson (255) expresses the antecedent of one of these counterfactuals concerning the seesaw as "if child i were moved through a vertical distance δ in a virtual manner".) Wilson emphasizes that "these guiding virtual work considerations can be immediately identified as a set of manipulation conditionals of the sort Woodward highlights in his researches" (255). What does Wilson mean by terming these conditionals "manipulationist" (265, 308) and "Woodwardian" (263, 266)? He is appealing to Woodward's (2003) account of the conditionals expressing the pattern of counterfactual dependence that is associated with a causal relation. According to Woodward, a causal explanation of a physical quantity Y conveys information about Y's systematic counterfactual dependence – about how Y would have been different under various "interventions" (with respect to Y) on one or more other quantities X_i. Roughly speaking, an "intervention" on X_i (with respect to Y) is a counterfactual operation that would "manipulate" X_i directly, resetting X_i's value without changing Y as a means of changing X_i and despite leaving unchanged X_i's actual causal antecedents and any of Y's causal antecedents not caused by X_i – so any change to Y as a result of the intervention must come from X_i's causing Y. Such an "intervention" may well be physically impossible (Woodward 2003: 128–33). That is no problem, as long as it is "possible" in a broader sense (in involving a conceptually possible, well-defined physical operation). Furthermore, under an intervention on X_i (with respect to Y), any relationship between the X_i and Y must be held fixed (in the way that laws of nature are preserved under counterfactual antecedents). However (Woodward 2003: 6), that relationship cannot hold for purely logical, conceptual, or mathematical reasons. (For example, the location of a system's center of mass would have been different under the displacement of one of the system's constituent bodies, but that displacement does not qualify as a "manipulation" of that constituent's location with respect to the center of mass because their connection is conceptual: the system's "center of mass" is *defined* as the average location of the system's masses.)

According to Woodward, an X_i helps to causally explain Y exactly when there exists such an "intervention" on X_i with respect to Y where, had it occurred, Y would have been different. For example (Woodward 2003: 197–8), a simple pendulum's length L can (partly) causally explain its period T because there is an intervention on L with respect to T (e.g., shortening the string by which the bob is suspended) where T would have been different, had L been different. (That was a "Woodwardian", "manipulationist" counterfactual since its antecedent was understood as positing L's difference as brought about by such an intervention on L.) By contrast, we cannot use T to explain L because there exists no intervention on T with respect to L where L would have been different, had T been different (as brought about by an intervention). Shortening the string and thereby causing L to change fails to qualify as such an intervention, since it changes L as a means of changing T. Moving the pendulum to a location with a different gravitational acceleration also fails to qualify as such an intervention, since had T been different (as brought about by this intervention), L would have remained unchanged. (In section 5, I will return to the asymmetric character of Woodwardian manipulationist counterfactuals.)

Whether the conditionals in the PVW's equilibrium condition should be interpreted as Woodwardian manipulationist counterfactuals (and why Wilson so interprets them) will be among the topics of sections 4 and 5. But first we need to look more carefully at what Wilson aims to demonstrate from examining the PVW.

3 The work that Wilson uses the PVW to do

Wilson regards his study of the conditionals we use in applying the PVW as supporting at least three closely interrelated morals. I will present these morals now and some of the arguments for them that might be extracted from Wilson's text. I will maintain that these arguments are insufficient. But Wilson has more weighty arguments to deploy; I will turn to them in section 4.

First moral: Wilson maintains that PVW counterfactual conditionals cannot be understood as invoking things as extensive as possible worlds; the counterfactual antecedents "stoutly resist ready enlargement into richer possible worlds" (311).

Wilson's reason for this view might initially be expected to involve the fact that the PVW "exploits higher-scale constraint knowledge" (311) where the laws holding the seesaw's parts together are "largely unknown" (263); "classical mechanics never settled upon a reliable set of special force laws adequate to material cohesion and a host of other factors" (57; cf. 312 fn. 42). But this consideration is inadequate to show that the macro-level antecedent of one of these "peculiar 'virtual work' counterfactual conditionals" (311) cannot be expanded to embrace a complete micro-level description of the world. Even if neither classical physics nor anything else we know supplies a micro-level account of the constraints, presumably there is some such account (not yet fully elaborated) that yields the constraints (at least approximately).

Though this argument fails to demonstrate this moral, I believe that Wilson's primary motivations for this moral go deeper. Those motivations appeal to the "virtual" character of the displacements posited by the conditionals in the PVW's necessary and sufficient condition for equilibrium. In the next section, I will turn to that argument.

Second, closely related moral: Wilson maintains that PVW counterfactuals require a highly specialized reading of the counterfactual antecedent and of what would happen under it, rather than a "one-size-fits-all" (263) standard "closeness-of-worlds metric" derived from a "general theory of counterfactual conditionals" (311). We care about such a "peculiar" metric only because its use yields accurate empirical predictions (310). Wilson intends this case to be one motivation for his broader critique of a "philosophical movement" that "calls itself analytic metaphysics and patterns itself after the proposals of the late David Lewis" (241) according to which "the tenets of metaphysics supply a conceptual pre-science upon which any properly developed science must later build" (242). Wilson's critique is roughly that such a general set of metaphysical categories that all scientific reasoning must employ fails to do justice to the multiple, disunified "strategic patches into which profitable reasoning frequently divides" (243), with "a bewildering array of explanatory architectures" (247). Any "permanent" (244) conceptual repertoire, intended by "analytic metaphysicians" to reflect "an absolute and timeless necessity behind our talk of 'parts', 'wholes,' and 'causes'" (244), "locks science within a conceptual straitjacket that fails to account for the subtle adjustments at the core of its improving practices" (278) and "reflects a philosopher's presumption that he or she can ably pronounce upon the 'basic structures of science' without bothering to study them in any detail" (280).

Third, closely related moral: Wilson maintains that PVW counterfactuals are not "grounded in laws" (257; cf. 265). This is connected to Wilson's opposition to a one-size-fits-all, permanent, metaphysically motivated repertoire of concepts; he believes that general accounts of counterfactuals portraying them as grounded in laws are unable to accommodate the "peculiar" PVW counterfactuals (262). Moreover, he opposes the category "laws of nature" as one of those allegedly fundamental, permanent categories beloved by analytic metaphysicians but failing to do justice to the variegated character of scientific practice (263; cf. 152).

Wilson's text suggests several readings of his idea that PVW counterfactuals are not "grounded in laws." One reading is epistemological: we can ascertain these counterfactuals without knowing the laws underlying the constraints; these counterfactuals were known in classical physics by straightforward induction on our observations of (e.g.) seesaws, while the laws making the seesaw's microstructure rigid were unknown.

It seems to me that this epistemological point, even if true, does not bear upon whether PVW counterfactuals are "grounded" in laws; laws could be partly responsible for making them true even if they can be known without knowing the laws. The order of knowing need not follow the order of being. Wilson conflates metaphysics with epistemology when he writes (referring to the PVW) that "closely analyzed scientific exemplars – like the carefully monitored variations of Lagrangian technique – … show that the truth-values of the restricted class of counterfactual possibilities exploited in Lagrange's manner commonly stem from direct induction upon manipulative experimentation, such as the tinkerings of our do-it-yourselfer" (316; cf. 257, 265). What gives the counterfactuals their truth-values cannot be immediately read off of how we ascertain those truth-values.

Wilson is presumably correct that we can understand and ascertain at least some counterfactual conditionals long before we have grasped the notion of a law of nature or discovered any relevant laws, and so neither in the order of conceptual learning nor in the epistemic order do laws come before all counterfactuals (266). But this fact does not arise from any peculiarity of PVW counterfactuals; it is true of many ordinary counterfactuals. (Each of us personally and science historically knew that the match would have lit, had it been struck, long before we knew the laws underwriting this fact – e.g., underwriting the fact that there would still have been oxygen present, had the match been struck.) In any case, neither the order of conceptual learning nor the epistemic order is the order of "metaphysical grounding" (262), which is the official subject of Wilson's claim that laws fail to ground PVW counterfactuals. Nevertheless, Wilson insists that "claiming – as [analytic metaphysicians] do – that the truth-values of these counterfactuals must be dependent upon the very laws that they are designed to circumvent strikes me as strange" (316). I see nothing strange in our knowing a fact without knowing its "ground", as when we knew that the puddle was water before we knew about water's molecular structure. Likewise, even if PVW counterfactuals are ascertained "directly … from experimental twiddling," rather than from "law-based speculation," this suggests nothing about whether PVW counterfactuals owe their truth-values to unknown microlaws, much less that an account of their truth-makers that proceeds by "embedding standard Lagrangian 'possibilities' within inflated possible worlds would immediately erase all … special efficiencies" (291) in problem-solving that the PVW provides.

In fact, I am inclined to doubt that our reasons for believing PVW counterfactuals "commonly stem from direct induction upon manipulative experimentation." That is because of the *virtual* character of the displacements posited by their antecedents. The virtual character of these posited displacements is the source of Wilson's primary and most powerful motivations for his morals. I will now turn to them.

4 The virtual character of virtual work

Wilson sees his morals as strongly suggested by the conditionals employed by the PVW: "The chief exemplar we shall discuss is the manner in which simple macroscopic counterfactuals are central to Lagrange's celebrated virtual work techniques…" (291). In particular, Wilson's arguments appeal to the *virtual* character of the displacement and work involved – which we will now examine. There are two readings of "virtual" that are commonly found in physics textbooks. Although I believe that only the second of these readings actually figures in Wilson's own arguments, both might appear to be capable of motivating his morals. Since virtual work and displacement have received relatively little philosophical attention (and since many physics textbooks skate lightly over these matters), I will discuss both of these readings of "virtual".

To appreciate the first reading, suppose that the system is in equilibrium with all of its components at rest. The PVW, as Wilson characterizes it, traffics in conditionals concerning the work that would have been done on the system, had the system undergone an infinitesimally small displacement in some particular manner allowed by the constraints. The system's equilibrium, according to the PVW, is reflected in the fact that had it undergone such a displacement, the non-constraint forces on it would have done no work. However, while the system is in equilibrium, it undergoes no displacement; that it is in equilibrium just means that all of its components would remain where they are, as long as the system begins from rest and the external forces on and intrinsic properties of the system remain unchanged. That the system undergoes some displacement, then, apparently contradicts the fact that the system is in equilibrium – the fact that the PVW is supposed to help show. Some textbooks state explicitly that "virtual" denotes this feature of the displacement and work, as in Spivak (2010: 179): "As for the word 'virtual' here, … it refers to the fact that although we have obtained [the equation that the total virtual work performed under the displacement equals zero] under the assumption that our rigid body is in equilibrium, we have done so by considering … motions that our rigid body might have had if it weren't in equilibrium."

It might seem that we have here a way to use PVW counterfactuals to motivate Wilson's three morals: The circumstances that PVW counterfactual antecedents posit "stoutly resist ready enlargement into richer possible worlds" (311) because a PVW counterfactual's antecedent is contradictory! It posits that a system is simultaneously at equilibrium and undergoing a displacement. A contradiction is impossible and so cannot be expanded into a possible world. To motivate Wilson's other morals, one might argue that perforce the antecedent contradicts the laws of nature, so the PVW counterfactuals cannot be grounded on laws. Furthermore, such an exotic counterfactual antecedent is surely not covered by some standard, off-the-shelf philosophical account of counterfactuals; the conditional is non-trivial only within a highly "peculiar" context. Indeed, some textbooks portray the posited displacement as contradictory, as in Feynman et al. (1963: p. 4-5): "This approach is called the principle of virtual work, because in order to apply this argument we had to *imagine* that the structure moves a little – even though it is not *really* moving or even *moveable*." That it is moving but not moveable certainly sounds contradictory. (See also Poinsot 1975: 107–8.)

It seems to me, however, that this is not Wilson's argument and, moreover, that there is in fact no contradiction in the counterfactual antecedent. The antecedent posits that the system moves in a certain direction – not that it moves while it does not move (or while it is not moveable or remains in equilibrium). There is no pressure at all to say that the closest possible world where the PVW conditional's antecedent holds is a world where the system is moving and not moving. Rather, the system is *actually* motionless but *counterfactually* in motion. There is no contradiction in taking the fact that the system would have done no work, had it been moving rather than in equilibrium, and then using that fact together with the PVW to entail that the system is actually in equilibrium.

Of course, the PVW counterfactual's antecedent does not specify the force, if any, that would have caused the posited displacement, had that displacement occurred. But we do not need to know anything about that force – even whether there would have been any such force – in order to evaluate the work that would have been done by the (other) non-constraint forces, had the system undergone this displacement. (I will say more about this below.) Indeed, one of Wilson's reasons for interpreting PVW counterfactuals as "Woodwardian", "manipulationist" counterfactuals is presumably that the counterfactual displacement is posited as brought about directly – by an intervention occurring without the usual causal antecedents. (As I mentioned in section 2, a Woodwardian intervention may even be physically impossible.) Like other Woodwardian interventions, how this departure from actuality is brought about does not matter; just as a Woodwardian intervention on the length of a pendulum's string posits the string's being shortened without considering the cause of that shortening, so the PVW "is concerned with the work associated with a small, arbitrary, *imaginary* displacement, the cause of which is not considered" (Charlton 1955: 139).

So far, then, PVW counterfactuals seem no different from other Woodwardian counterfactuals. If PVW counterfactuals are to motivate Wilson's morals, then (unless many other Woodwardian counterfactuals could do so just as well) this motivation must be some feature of PVW counterfactuals that distinguishes them from other Woodwardian counterfactuals. I am inclined to think that Wilson sees this key feature of PVW counterfactuals as contributed by the second reading of *virtual* that is commonly found in textbooks. This understanding of virtual displacement and work (to which I will now turn) seems to be what motivates one of Wilson's principal arguments for his morals.

In the course of the counterfactual displacement, the actual non-constraint forces and the actual constraints are held fixed (as Wilson emphasizes on pp. 309–11 and 169). For example (see Fig. 5), suppose a body lying on a flat, frictionless floor is attached to a spring, the other end of which is attached to an immovable wall. Suppose the spring is at its equilibrium length, so it exerts no restoring force. A displacement allowed by the constraints (e.g., that the body remains attached to the spring and on the floor) consists of the body's being pulled so that the spring is stretched beyond its equilibrium length. Although the restoring force increases from zero in the course of that displacement, *the force used in calculating the virtual work remains zero*, since zero is the actual force. (That the force is zero makes the virtual work under this displacement equal zero, so the PVW gives the correct answer: that the actual system is at equilibrium.) In addition, under the displacement, the constraints are also held to their actual values. For example,

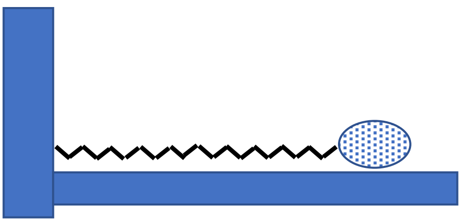

Fig. 5 A body attached to a spring and constrained to lie on the floor

suppose the spring apparatus is constrained to lie on the floor of an ascending elevator. Although the elevator floor is actually increasing its z coordinate, the floor (and hence the body) remains at constant z during a virtual displacement; such a displacement moves the body in the x or y directions, but at constant z.

Accordingly, the constraints and non-constraint forces are often said by textbooks (e.g., Bhatia 1997: 21; Calkin 1996: 40; Hamill 2010: 503; Humar 2002: 181) to be "frozen" over the course of a virtual displacement. Since constraints and non-constraint forces may actually change over time, some textbooks say that a virtual displacement takes place outside of time or while no time (not even an infinitesimal dt) elapses (e.g., Planck 1933: 168). Textbooks typically use a lower-case delta rather than "d" (that is, "$\delta \mathbf{s}$" rather than "d$\mathbf{s}$") in expressing virtual work and displacement "to distinguish them from normal differentials", that is, to mark "the convention that the time t is held fixed" (Johns 2011: 47). (Wilson follows this practice.)

This respect in which the displacement and work are virtual might seem to support Wilson's morals. The PVW counterfactuals' antecedents, on Wilson's view, "must include the supplementary qualifier 'virtual.' That is, they should assume the form, 'if bead b_i is *virtually* lowered through a distance δq…'" (309). Therefore (I take Wilson to be arguing), the counterfactual antecedent specifies that the constraints and non-constraint forces during the posited displacement are fixed to be as they actually are. Hence, no laws actually governing the constraints and non-constraint forces are needed to make the conditionals true; simply the actual values of the constraints and non-constraint forces, not laws specifying how their values depend on other circumstances (such as the spring's length), are what the truth of PVW conditionals rest on. I am inclined to take this to be at least one of Wilson's principal reasons for saying that "it is easy to see that present considerations [concerning the difference between real and virtual work] strongly bolster the argument against the advocates of 'counterfactual grounding'" (310 fn. 39), supporting one of Wilson's morals.

Moreover, the PVW counterfactuals' antecedents posit violations of the actual laws (such as springs being stretched while their restoring forces remain fixed). Indeed, as we have seen, textbooks refer to virtual displacements as occurring while time itself is not passing! A counterfactual antecedent positing such exotic circumstances would presumably not be dealt with by standard, off-the-shelf accounts of counterfactual reasoning – another of Wilson's morals. Furthermore, that virtual displacements take place outside of time may well be one of Wilson's reasons for regarding the virtual character of these displacements as "the particular aspect of Lagrangian technique that clearly indicates the follies of inflating useful local possibilities into gargantuan possible worlds" (309; cf. 302, 311) – another of his morals.

If (as I am inclined to think) this is at least one important way that Wilson sees his three morals as motivated by the virtual character of PVW displacements, then I do not find these motivations to be very powerful. As we have seen, Woodwardian manipulationist counterfactuals can posit interventions that violate the actual laws. The same goes for a typical counterfactual on Lewis's account (an account that Wilson excoriates as unable to accommodate the hothouse environments required by delicate PVW conditionals). According to Lewis's account of counterfactuals, a "small miracle" (a single, localized violation of actual laws) is standardly understood as bringing about the event posited by the counterfactual antecedent. That a counterfactual antecedent is understood in this way does not suggest that the counterfactual posits so exotic a circumstance as to place the counterfactual outside the scope of a standard semantics of counterfactual conditionals.

Furthermore, just as PVW counterfactual antecedents posit that the various actual constraints and non-constraint forces are held fixed at their actual values, so various Woodwardian counterfactuals expressly posit that various causal intermediaries in various causal chains are to be held fixed. Only by holding the right things fixed in a given case can Woodward make the counterfactual dependence of one event on another correspond to their causal dependence. A Woodwardian counterfactual that reflects whether or not there exists some particular causal path from *X* to *Y* posits a manipulation of *X* that holds fixed not only *X*'s actual causal antecedents, but also any of *Y*'s causal antecedents not caused by *X* via that particular path – so that any change to *Y* as a result of this intervention must come from *X*'s causing *Y* via the path in question (Woodward 2003: 54–55, 136).[3] Thus, if PVW conditionals qualify as "peculiar" because of their expressly holding various facts fixed, then so do many Woodwardian manipulationist conditionals.

I agree with Wilson that a PVW counterfactual's antecedent specifies that the constraints and non-constraint forces are fixed under the posited displacement to be as they actually are, and that therefore a PVW conditional is not made true by laws specifying how the constraints and non-constraint forces would have been different under various counterfactual circumstances. Rather than those laws, the actual values of the non-constraint forces are among the PVW counterfactuals' truthmakers. But this fact provides little support for any moral that the category of natural law plays no role in PVW reasoning. Apart from the fact that the PVW itself is a natural law, a given system's actual constraints and non-constraint forces are explained partly by various laws governing the system's internal constituents. Knowledge of laws may even be needed to ascertain the actual values of non-constraint forces, though this is epistemology, not metaphysics.

Of course, Wilson has laid the groundwork for disputing the power of all of these considerations.[4] He would presumably regard the idea that there are laws (some as yet undiscovered) governing the system's internal constituents and explaining the system's actual constraints and non-constraint forces as "mistaking aspirational hope for accomplished task … upon some nebulous a priori basis" (78). Although background knowledge may be needed to ascertain the actual values of a system's non-constraint forces, that this background knowledge should be interpreted as knowledge of *laws* is (Wilson would presumably object) unsupported by scientific practice, but instead motivated by "loose thinking of a Theory T stripe" common among "contemporary analytic metaphysicians" (78). All of this moves the dispute to general issues well beyond this paper's scope – broad issues that do not concern the PVW specifically.

There is a closely related but somewhat different way to use the virtual character of the displacement posited by PVW conditionals to argue that "our virtual possibilities stoutly resist ready enlargement into richer possible worlds" (311). For at least some systems, it is true that had the system undergone a virtual displacement, then the

[3] For example, Woodward (2003: 136) considers a structure where there is a direct causal route from *X* to *Y* and an indirect route from *X* through *Z* to *Y*. To reveal the direct causal dependence of *Y* on *X*, Woodward says, we need a counterfactual about how *Y* would have been different had *X* been made different (by an intervention) while (by another intervention) *Z* was held fixed.

[4] My thanks to a referee for pressing these points.

constraints and non-constraint forces (being fixed as they actually are, rather than changing as the actual laws require) would have to have been governed by laws different from those that actually govern them. It is unclear what those laws would have been. There is no guarantee that those laws would have permitted anything like the microstructures that the system's bodies actually have. The microstructures allowed by those laws might not even have permitted stable macroscopic bodies to exist. I also see no way to guarantee that a set of laws could allow the non-constraint forces to remain frozen under the displacement, contrary to actual law, while also fixing the constraints to be as they actually are. For instance, if (returning to my earlier example) a spring's restoring force would have remained unchanged under the virtual displacement of the body attached to the spring, then perhaps the internal "springs" that connect the body's molecules (and the floor's molecules) would likewise have behaved differently than in the actual world – in which case the constraints that depend on the body's (and floor's) rigidity would presumably not have remained as they actually are. If we must take into account these indefinitely ramifying consequences of the posited displacement's virtual character, then PVW conditionals will be impossible to think through. But there is no reason for us to take these consequences seriously unless we go beyond the PVW's limited concerns to ask about the laws and microstructures that would have obtained under the posited displacement (which a standard, off-the-shelf closeness-of-worlds metric might encourage us to do). These consequences arise, in other words, only if we try to "enlarge" the antecedent into an entire world. These consequences never arise in the scientific practice of applying the PVW.

Although Wilson does not (as far as I can tell) give an argument along precisely these lines, he does seem to be concerned generally with such obstacles to building an entire possible world around a virtual displacement. (In any case, his remarks may lead his readers to entertain such an argument.) Such an argument seems to me a promising option for deriving Wilson's morals from the virtual character of the posited displacement. Nevertheless, this approach does not fit very well with Wilson's characterization of PVW conditionals as "peculiar" (262) or the contexts in which they function as special. The above argument's applicability is not confined to conditionals involving virtual displacements or even to conditionals drawn from Wilsonian technical contexts in mechanical engineering and applied physics. On the contrary, even for many ordinary counterfactuals, the very same argument can be made for thinking that they function only in contexts where it is inappropriate to entertain ways of "inflating useful local possibilities into gargantuan possible worlds" (309).

Take an example familiar from the philosophical literature.[5] Suppose that Mr. Darcy and Elizabeth Bennett actually quarreled two days ago, and Elizabeth was then so cross that had Darcy asked Elizabeth for a favor yesterday, she would not have granted it. Standard semantics for counterfactual conditionals portrays this counterfactual's antecedent as directing our attention to a possible world where Darcy asks Elizabeth for a favor but where until he does so, the events that transpire are the same as in the actual world. If the actual laws are deterministic, then Darcy's action in making his request (given the history to that time) violates the actual laws by occurring without any of the causal antecedents the laws require – "miraculously", as Lewis says. But if (as standard

[5] This example originates with Downing (1959) and was popularized by Bennett (1974) and Lewis (1986:33-34).

semantics alleges), had Darcy requested a favor, the laws of nature would not have been the actual laws, then the laws might well have been different in other ways that must be taken into account in determining what would have happened had Darcy requested a favor (just as had Coulomb's law been violated by a given pair of uniformly charged spheres, then Coulomb's law might well have been violated by other such sphere pairs as well – and by uniformly charged plates and other things, too). If the laws would have been different, had Darcy requested a favor from Elizabeth, then those different laws might even have led to Elizabeth's granting Darcy's request. Now that we have started thinking along these seductive lines, the consequences of the counterfactual posit ramify indefinitely. Had Darcy asked Elizabeth for a favor yesterday, then after she rebuffed him (presuming she would have), wouldn't he have wondered how he came to make such a request in the first place? Did the prior quarrel somehow slip his mind? Or did he forget his pride? Wouldn't Darcy become concerned about whether he will engage in additional erratic, heretofore uncharacteristic behavior – and about whether others will do so?

Of course, these are bizarre questions that even a child would not ask because even children understand how ordinary counterfactual thinking operates. In the familiar sort of conversational context where it is true that Elizabeth would not have granted Darcy's request (had he asked her for a favor yesterday), the events causing Darcy to make his request are simply not under consideration. A participant in the conversation should and would start to consider them only if the context changes drastically. We philosophers start to consider them only if we are impelled by some philosophical theory to regard the counterfactual antecedent as invoking an entire possible world. But in the ordinary sort of context where "Had Darcy asked Elizabeth for a favor" gets its familiar meaning, a conditional about how Darcy came to make his request (or later wondered about how he came to do so) makes no sense. Counterfactual conditionals generally depend for their meaning on a surrounding context – but in this case, the context fails to supply such a conditional with what it requires in order to express any proposition at all.

Bennett (2003: 284-5) says that some counterfactuals (forming a "mildly degenerate but quite common kind") involve "no thought about a possible history for the antecedent," as when "Charles's wife remarks sarcastically, 'If Charles had been CEO of Enron, the accounting fraud would not have lasted a week' because Charles is incompetent with money." Bennett says that Charles's wife in this context would regard "as an irrelevant nuisance the question of whether Charles could have *come* to run Enron while still financially incompetent." In the same way, it is an irrelevant nuisance in a PVW context how the laws governing the constraints and non-constraint forces would have been different, had a given virtual displacement occurred, and what differences those differences would have made. I agree with Bennett regarding his example, except I regard the example as perfectly typical rather than "a marginal and uninteresting sort of counterfactual" (Bennett 2003: 255).[6]

If this is Wilson's line of thought, then (like Bennett) he undersells it. I agree with Wilson in regarding any counterfactual conditionals employed by the PVW as having to invoke something much narrower than an entire possible world. But to arrive at this moral, we need not reach for anything as "peculiar" as PVW conditionals; the same moral is illustrated by many ordinary counterfactuals.

[6] I have argued more fully for this view in Lange (2009: 200-205).

However, I will now suggest that it may be better to regard the PVW as involving no counterfactual conditionals at all.

5 Are PVW conditionals counterfactuals?

As we have seen, Wilson says that "these guiding virtual work considerations can be immediately identified as a set of manipulation conditionals of the sort Woodward highlights in his researches" (255), and from this identification he draws his morals. I have just explored the prospects of various arguments for drawing those morals from interpreting the PVW as using counterfactual conditionals to specify necessary and sufficient conditions for equilibrium. However, there are at least two reasons to wonder whether it is appropriate to interpret PVW conditionals as Woodwardian manipulationist counterfactuals in the first place.

First, PVW conditionals are unlike the manipulationist counterfactuals that Woodward associates with causal relations because (as we saw in section 2) Woodward specifies that his conditionals cannot hold for purely logical, conceptual, or mathematical reasons. By contrast, the relation between the force applied to a body, the distance the body moves under that force, and the work done is conceptual, not causal. "Work" in mechanics is (as we saw in section 2) simply shorthand for a vector-algebraic combination of force and distance. Of course, it is not a conceptual fact that the body attached to the spring (in my earlier example) experiences zero restoring force from the spring. But interpreted as a counterfactual conditional, the PVW conditional in this example is "Had the body been *virtually* displaced to the right by a distance δx, then zero virtual work would have been done." If "virtually displaced" in this case is understood as specifying not only that all of the constraints (e.g., that the body remain on the table) are respected, but also that the restoring force exerted by the spring is fixed at its actual value of zero, then this conditional is a purely conceptual truth.

Of course, that the PVW conditional is conceptual fits well with one of Wilson's morals: that the conditional does not depend for its truth on any laws of nature. Wilson might therefore welcome the conclusion that the conditional is conceptual, even though it also makes the conditional dissimilar to Woodward's manipulationist conditionals. To regard this conditional as tracing a *causal* relation would be mistaken for the same reason as (for example) it would be a mistake to regard a given student's weight as one *cause* of the average weight of the students in the class.

On the other hand, Wilson might reply that a further moral of PVW considerations is that counterfactuals that are purely conceptual can nevertheless trace causal relations. After all, Wilson maintains that "any rigid requirement on how 'causes' must relate to 'effects' is apt to crumble as these words naturally guide themselves into local architectures in which distinctive bonds between words and world can subsequently solidify, in accordance with the practical objectives advanced by those local employments" (259). Perhaps Wilson would insist that once we "attend to altered forms of 'causal relationship' in the context of varied explanatory architectures" (315), we will conclude that Woodward's restriction of manipulationist counterfactuals to conditionals that are not merely conceptual unduly restricts scientific progress: "the pragmatic factors that drive language relentlessly onward cannot be pestered by the annoying necessities of the philosophers" (244).

But this response seems too permissive. We should insist on distinguishing causal relations from the conceptual relation between (e.g.) a system's mass distribution and the location of its center of mass (even though the center of mass's location can be changed by changing the location of one of the system's bodies). The latter is like the conceptual relation between force, distance, and work. I would insist on this distinction not on some "nebulous a priori basis" (78) but rather on the grounds that this distinction is incorporated into scientific practice. For instance, when a 5 kg body moves to a location 5 m away from me, I instantly acquire the property of being 5 m away from a 5 kg body. But I would insist that my acquiring this property was not *caused* by the body's moving to a certain location 5 m away, on pain of action at a distance on the cheap. Scientists who were concerned about action at a distance did not have (and should not have had) their concerns allayed by this case. Perhaps this case is disqualified from being causal on the grounds that the relation it involves is conceptual. Perhaps instead it is disqualified from being causal on the grounds that none of my intrinsic properties was involved. Whatever the correct account may be (Kim 1974; Lewis 1986: 272–4), that account holds fast to some essential features of causal relations rather than, "once a specific explanatory landscape has been selected", allowing "'cause' [to] enter the new territory and claim some of its salient landmarks as its own (rather as the presumptuous chair of a search committee might award the job to himself)" (77).

There is a second, related reason to be hesitant about immediately identifying PVW conditionals as Woodwardian, manipulationist conditionals. Woodward's conditionals are asymmetric, as befits conditionals tracing (asymmetric) causal relations. We saw an instance of this asymmetry in the pendulum example at the end of section 2. By contrast, PVW conditionals can run in both directions. For instance, in section 4's case of the spring in the elevator (Fig. 5), scientists can calculate from the zero restoring force (which is held fixed under virtual displacement) that the virtual work that would have been done under each virtual displacement would have been zero, and then from the PVW, it follows that the system is in equilibrium. But scientists can also reason in the other direction: they can ascertain what the actual force on a given system would have to be, in order to bring it into equilibrium, by using the PVW to conclude that in equilibrium, zero virtual work would have been done under a given displacement, and then using a conditional such as (in the spring example) "Had zero virtual work been done in virtually displacing the body to the right by a distance δx, then the net non-constraint force on the body in the x direction would have been zero." Scientists can then conclude (from the fact that the restoring force is the only non-constraint force operating) that the restoring force must be zero. (Of course, the answer will not always be zero; in cases like those in section 2 involving pulleys, the PVW can be used in this way to figure out how hard someone has to tug on the end of the rope in order to equilibrate the system.) That the PVW conditional traces a conceptual rather than a causal relation enables it to be symmetric.

That PVW conditionals cannot be "immediately" identified as Woodwardian, manipulationist conditionals might lead us to consider another interpretation of them. Rather than interpreting the PVW as concerned with the virtual work that would have been performed, had certain virtual displacements occurred, we might instead interpret the PVW as specifying that a system is at equilibrium iff a certain mathematical operation on the system's quantities yields zero. That operation, of course, involves considering infinitesimal displacements allowed by the constraints and taking the sum $\sum_i \boldsymbol{F_i}\cdot \mathrm{d}\boldsymbol{s_i}$ for each such displacement and the actual non-constraint forces $\boldsymbol{F_i}$. The PVW could then be expressed in terms of an algebraic combination of physical quantities rather than a subjunctive conditional concerning hypothetical displacements: If the $\boldsymbol{F_i}$ are the actual non-constraint forces and the $\mathrm{d}\boldsymbol{s_i}$ are infinitesimal displacements allowed by the constraints, then equilibrium obtains iff $\sum_i \boldsymbol{F_i}\cdot \mathrm{d}\boldsymbol{s_i}$ vanishes for all permissible displacements. No morals about counterfactuals, possible worlds, laws, and so forth then follow from the "frozen" character of the constraints and non-constraint forces; the requirement that we use their actual values in this procedure is simply part of what Wilson calls the "computational architecture" (257). Although this "reliable inferential practice" (263) involves summing the products of non-actual displacements with actual forces and hence involves summing quantities having the dimension of work, it is misleading to think of these quantities as the work that would have been done, had certain infinitesimal displacements occurred.

Textbooks sometimes explicitly contrast the "frozen" constraints and non-constraint forces employed by the PVW with genuinely counterfactual matters: the constraints and non-constraint forces that would have obtained, had certain (non-virtual) infinitesimal displacements occurred. For example, Neal (1964: 41) says that in the PVW procedure, "it is not necessary … for the bar forces to be those that would be induced by the loads." Neal thus devotes counterfactuals (even in a PVW context!) to displacements where the constraints and non-constraint forces are not frozen. He thereby motivates an interpretation of the PVW that involves no counterfactuals about what would have happened, had certain displacements occurred. Neal says that to regard the forces figuring in the PVW as the forces that would have occurred under certain circumstances may be "a useful aid to memory" (39) but "it is inadvisable to attach even this spurious degree of physical meaning to the principle" (38). Rather, the PVW prescribes a computational procedure involving "mathematical experimentation in which the system is given the virtual displacement" (Humar 2002: 181).

That the PVW concerns what the output of a certain mathematical procedure on the system's quantities is, not what the outcome of a certain sort of displacement would have been, is also encouraged by a thought that we have already seen to be frequently found in textbooks: that a virtual displacement occurs outside of time. Capecci (2012: 1, 10, 339) contrasts these two interpretations, arguing that both have been given in the course of the PVW's history:

> Since the Greek origins of mechanics, there have been two alternative formulations of laws of virtual work…In the early days of VWLs, virtual motions were considered primarily as possible motions, those which one would have imagined the body, or system of bodies, to assume within the respect of constraints, for example, following a disturbance induced by a small force that alters the equilibrium. … But with this type of "natural" conception there coexists another … in which the virtual motion is seen as purely geometric … with a time flowing in a super-celestial world. This way of viewing virtual motions began to emerge … to become the "natural" one only in the XIX century with Poinsot and Ampère. … According to Poinsot real time and virtual time run on different universes:

> It must be further noted that the system is supposed to move in any way, without reference to forces that tend to move it: the motion that you give is a simple change of position where the time has nothing to do at all.[7]
> …One cannot stress enough the fact that Poinsot's virtual velocity is purely geometric and his virtual work is only a mathematical definition. … [Poinsot contends] that virtual velocities refer to changes of position occurring in a virtual time while the real time is frozen…Towards the end of his text Poinsot writes "It would therefore be futile to search for the metaphysics of the principle of virtual velocities and to endeavor to understand what they are in themselves…"[8]

I see no reason to interpret the PVW as involving counterfactual conditionals rather than a mathematical procedure.[9] Of course, on the latter interpretation, no possible worlds are invoked and no laws governing the constraints or non-constraint forces are needed for the PVW to apply, just as Wilson says. But no morals about the grounding of counterfactuals are bolstered.

6 Conclusion

I have now reviewed various ways that the PVW might be interpreted and various ways that Wilson might be trying to harness the PVW in support of his morals. Although I am sympathetic to some of these morals, I have not found the PVW to be especially well suited to support any of them. There is one further aspect of the PVW that I have not examined and that Wilson occasionally mentions: the PVW's explanation, i.e., the reason why it holds. I will conclude by marking it as deserving closer scrutiny.

Wilson says, "If we turn Lagrangian techniques on their head in that grounding-in-laws manner, we won't be able to understand the reliability-enhancing utilities of this popular form of scientific reasoning at all" (316). Perhaps Wilson's point is that if we take PVW reasoning as employing knowledge of the system's microstructure, internal macroscopic mechanism, or laws governing non-constraint forces, then we will fail to understand why the PVW is so useful. This is correct since (as we have seen) the PVW does not draw upon any such knowledge – which makes the PVW useful, since oftentimes we lack such knowledge. But Wilson may be concerned here less with the PVW's usefulness, considering our limited knowledge, and more with the PVW's explanation: "to understand" the reliability of PVW reasoning may be to give a scientific explanation of its yielding accurate predictions from accurate premises. Why does the PVW obtain?

Perhaps it is explained "from the bottom up": by separate laws, each of which governs one of the simple machines (e.g., lever, pulley, wheel) or elementary causal

[7] "…où le temps n'entre pour rien" (Poinsot 1975: Appendix p.13).

[8] This passage appears shortly after the passage in note 7. As Wilson notes (p. 169 fn. 51), Duhem (1980: 114) says that "The use of these virtual modifications is an artifice of reasoning, a calculational process; it is therefore useless for a virtual modification to have a physical meaning." But Duhem's interpretation of the PVW is bound up with a broader instrumentalism.

[9] That the PVW concerns a mathematical procedure does not deprive it of physical content, since on this interpretation, the PVW specifies that the mathematical procedure applied to the system's physical quantities has a certain outcome iff the system is in equilibrium.

processes. If each of these machines or processes obeys the PVW for its own separate reason, then any combination of them will, too, and hence the PVW will obtain generally. On this view, the fact that the PVW holds for one system does not generally have the same explanation as the fact that the PVW holds for another system; that it holds for them all is a sort of coincidence (albeit physically necessary). On this view, although the PVW allows us to *infer* that two physically disparate systems are both in equilibrium, it does not itself constitute a common *explainer* of their equilibrium. Rather, for each system, the various kinds of forces on (or force laws governing) it, together with the laws of motion, explain both why the PVW holds of that system and why the system is at equilibrium. As Whewell (1874: 333) says in endorsing this view, the PVW "serves verbally to conjoin Laws … [rather] than to exhibit a connexion in them: it is rather a help for the memory than a proof for the reason."

On the other hand, perhaps the PVW is explained "from the top down": by (for instance) general considerations arising from energy conservation.[10] In that case, the fact that the PVW holds for one system has the same explanation as the fact that the PVW holds for another system, despite their physical differences; it is no coincidence that the PVW holds of both. On this view, the PVW is a common explainer of the fact that various systems, involving different kinds of forces, are in equilibrium. The PVW does not depend on the particular forces operating in various systems; it would still have held even if there had been additional kinds of forces in nature. By contrast, if the PVW is a coincidence, then had there been additional kinds of forces in nature, the PVW might not still have held. Accordingly, Whewell emphasizes that the PVW "was established as a general principle by being proved in each particular case" (1832: viii) – that is, "by shewing its truth in the case of each of the mechanical powers separately" (1832: 22) – rather than by our inferring inductively that since the PVW holds of certain forces and mechanisms, it probably holds of the rest.

If the PVW is explained "from the top down", then we would be mistaken if we regarded the PVW as explained "from the bottom up" – as depending on the various force laws and other microlaws. Wilson (e.g., 265) appears to characterize the PVW as explanatory, not merely predictive; he characterizes the PVW as among the "bewildering array of explanatory architectures" (247) in physics. But Wilson offers no metaphysical account of how the PVW manages to explain – of what it needs to "stand on its own, independent of any detailed theory of … forces" (as Weinberg (1992: 158) describes symmetry principles as doing). To do so would presumably involve precisely the kind of "analytic metaphysics" that he repudiates.

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[10] In Lange (2017), I discuss the contrast between "top-down" and "bottom-up" explanations and the way that conservation laws explain.

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