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How Can Instantaneous Velocity Fulfill Its Causal Role?

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From the time of Zeno, instantaneous velocity has puzzled natural philosophers. Today, physics and mathematics textbooks routinely follow Bertrand Russell in explicating a body's instantaneous velocity not merely as *equal* to, but moreover as *identical* to the time-derivative of its trajectory. This approach exploits the conceptual advances in "arithmeticizing" the foundations of the calculus that were made over the course of the nineteenth century. On this view, a body's instantaneous velocity is ontologically parasitic on its trajectory; the property of having at time t an instantaneous speed of, say, 5 centimeters per second is nothing over and above the property of having a trajectory, in the neighborhood of t, possessing a certain mathematical feature.

Recently, this "reductive view" of instantaneous velocity has been challenged by some philosophers, such as Tooley (1988), Bigelow and Pargetter (1990), and Carroll (2002). Tooley, Bigelow, and Pargetter propose instead that a body's instantaneous velocity at t stands on an ontological par with and is metaphysically independent of the body's trajectory in t's neighborhood. A body's velocity joins its mass and electric charge as among its ontologically primitive properties. According to "velocity primitivism," instantaneous velocity and trajectory are related only by virtue of natural law, not by metaphysical necessity.

I shall look at some objections to the reductive view, paying particular attention to whether instantaneous velocity and instantaneous acceleration, as understood reductively, can play the causal and explanatory roles that classical physics is often interpreted as demanding of them. I shall argue that they cannot. Velocity primitivism, on the other hand, fails to recognize that instantaneous velocity is not merely nomologically connected to trajectory, but rather is essentially something to do with trajectory. To capture velocity's essentially kinematic character along with velocity's causal and explanatory roles, I shall advance a radical proposal: that instantaneous velocity is roughly akin to a dispositional property.

I shall confine myself to classical physics with (for the sake of simplicity) absolute space and time. (My remarks could easily be extended to relativistic physics. Quantum mechanics is quite another matter.) My aim is to give an analysis of instantaneous velocity and acceleration that respects the causal and explanatory roles traditionally ascribed to them in classical physics.

According to classical physics, every body has a definite position, velocity, and acceleration at every instant at which it exists. In particular, bodies in classical physics move in continuous trajectories; a body does not move from place to place without passing through the intervening space, and a body does not disappear at one moment and reappear only after some finite period of time has elapsed. A body's motion in classical physics is affected by outside influences (force fields, collisions) that cause the body to feel forces. The force that a body feels at a given moment affects the body's instantaneous acceleration a at that same moment according to Newton's second law of motion, $F = ma$, where F is the net force on the body and m is the body's mass. The various instantaneous accelerations that the body undergoes over the course of some period of time cumulatively change the body's instantaneous velocity from what it was at the start of that period. The body's velocity (that is, the speed and direction in which it is moving), in turn, causes the body to change its location in a certain way—that is, to follow a certain trajectory. In other words, the body's instantaneous velocity at a given moment is a cause of the path that the body subsequently takes.

Of course, a philosopher who believes that causal relations are not objective features of reality, or that it is not the business of scientific theories to describe causal relations, might regard it as no defect in the reductive account that it portrays classical instantaneous velocity and acceleration as unable to play the above causal roles. (Russell himself had a low opinion of causation, famously terming it "a relic of a bygone age" (1917, 80)). Moreover, even if a philosopher believes in the objective reality and scientific relevance of causal relations, she might nevertheless insist that classical mechanics should not be given a causal interpretation or that its correct causal interpretation portrays instantaneous velocity and acceleration as epiphenomenal rather than as playing causal roles. I shall not address these views directly. My task is merely to investigate what classical instantaneous velocity and acceleration would have to be like in order for them to play the causal and explanatory roles that classical physics is traditionally interpreted as attributing to them. On that interpretation, any difference between a body's location at one moment and its location at some later moment must have been caused by the body's having non-zero velocity at various intervening moments, any difference between a body's velocity at one moment and its velocity at some later moment must have been caused by the body's undergoing non-zero acceleration sometime in the intervening period, and every acceleration is caused by a force.[1] Naturally, one way to argue that instantaneous velocity is epiphenomenal under the proper causal interpretation of classical physics, or instead to begin to argue that classical physics should not be interpreted causally at all, is to argue that there is no plausible account of classical instantaneous velocity that permits it to play the causal role that it has traditionally been assigned. I shall argue that this is not so.

My concern with motion in classical physics might seem rather narrow. It might be insisted that there are ordinary, pretheoretic notions of instantaneous velocity and acceleration that are largely independent of classical physics, and that these notions should be our concern. However, it is not entirely evident to me that there really are any such clear notions. Consider an example.[2] The rise of classical physics brought with it a change in how the moon was described as moving, though not because of any change in what the moon's path through space was believed to be. Scientists employing the ancient theory of the moon (as lodged in a solid geocentric celestial sphere) characterized the moon as non-rotating as it keeps the same face toward earth.[3] Newtonians, by contrast, described the moon as rotating once in the course of each revolution around earth. It is not evident to me that we should regard these two views as employing some common, well-defined, pretheoretic notion of rotation.

Another reason why my concern with motion in classical physics is not overly narrow or outdated is that classical instantaneous velocity is unlikely to be an idiosyncratic notion. Whatever treatment we give it is likely to carry over, mutatis mutandis, to the instantaneous rates of change of many continuously changing quantities: the rate at which the water level in our bathtub is rising, the rate at which the temperature of the water in our teakettle is rising, the rate at which a balloon's volume is increasing as it is being inflated (or as the temperature of the air within it is rising), the rate at which the electrical resistance of a wire is increasing, the rate at which an ecosystem is fixing energy in photosynthesis, the rate at which your impatience is rising—perhaps even the rate at which the national debt is increasing. These instantaneous quantities, though differing in many respects, seem to play causal roles analogous to those played by classical instantaneous velocity. As we will see, the issues that I shall be addressing in connection with classical instantaneous velocity must be faced in connection with many other instantaneous rates of change.

Furthermore, questions arising from the proper interpretation of classical instantaneous velocity bear upon many broader metaphysical concerns. On Russell's reductive view, a body's classical instantaneous velocity at a given moment is not one of the body's intrinsic properties at that moment (or even a component of the world's state at that moment). But as we shall see, to term it a relational property of the body is potentially misleading, in view of its character as a limit. In addition, if (as I shall argue) classical instantaneous velocity is something like a dispositional property, then the world's instantaneous physical state (at least as depicted by classical physics, which is not supposed to be as weird as quantum mechanics) does not consist entirely of instantiations of categorical properties. If facts about instantaneous velocity are ontologically primitive and irreducibly subjunctive in a possible world operating according to classical physics, then there are more than categorical facts at the bottom of that world, and certain subjunctive facts there help to causally explain certain categorical facts there. (And if even a world operating according to classical physics cannot do without subjunctive facts at the bottom, then we should not be altogether surprised if the same applies to an irreducibly chancy quantum world.)

1. Russell's Reductive Account and the Causal Explanation Problem

In *Principles of Mathematics*, Russell set out the reductive view of instantaneous velocity. He proposed the "at-at" theory of motion: that "motion consists *merely* in the occupation [by the same body] of different places at different times" (1903/1937, 473). To distinguish fast motion from slow, Russell sensibly appealed to the limit of average velocities. A body's average velocity v over a given interval $[t_1, t_2]$ is the ratio $[x(t_2) - x(t_1)] / [t_2 - t_1]$, where x(t) is the function (the body's "trajectory") specifying the body's position at time t.[4] A body's instantaneous velocity at t_0 is what its average velocity in an interval surrounding t_0 tends towards as that interval becomes smaller and smaller. That is,

$$v(t_0) = \lim_{\Delta t \to 0} [x(t_0+\Delta t) - x(t_0)]/\Delta t$$

where the notion of a limit was made rigorous by Cauchy and Weierstrass:

$\lim_{x \to a} f(x) = L$ if and only if for each positive number ε, there exists a positive number δ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$.

In other words, no matter how near to L you dare f(x) to go (as long as you do not require the gap ε between them to become zero), f(x) meets the challenge for every x throughout some neighborhood (of width 2δ) surrounding a.

On this view, there is nothing more to a body's having instantaneous velocity v at t_0 than its trajectory's having v as its "time-derivative" at t_0. Intrinsically, two bodies may be no different at t_0 even though one is moving at t_0 with non-zero velocity v whereas the other at t_0 is at rest. Only the *relations* among one body's positions at the various moments in a neighborhood of t_0 differ from the *relations* among the other body's positions at the moments in that neighborhood.

Russell says that his reductive view involves the "rejection of velocity and acceleration as physical facts (i.e., as properties belonging *at each instant* to a moving point, and not merely real numbers expressing limits of certain ratios)" (1903/1937, 473; cf. Russell 1917, 84; Salmon 1980, 41 and 1984, 152). To understand what this means, let's compare velocity to another quantity from classical physics: the electric field.[5] A given point in spacetime is associated with a vector E that determines the electric force F felt by any point body there with electric charge q: $F = qE$. The vector E, in turn, equals the vector sum of various contributions made by other charged bodies. In a static case, for instance, to E at a distance r in a certain direction from here, a point charge q here contributes a vector in that direction of length (q / r^2). Among the possible interpretations of E, I mention two:

(i) E describes a property of an extended, immaterial entity (the "electric field") that exists on an ontological par with matter; the electric field's strength and direction at a given spatiotemporal location are primitive properties of the electric field just as mass and electric charge are primitive properties of a body.
(ii) E stands for nothing more than the result of a certain calculation (for example, a vector sum of various (q / r^2)'s) involving the properties of distant charged bodies.

On the first interpretation, E describes a condition of the electric field that is caused by distant charges and able, in turn, to cause a charged body to feel a force (in accordance with $F = qE$). The electric field is a local cause of the electric force on a charged body. In contrast, the second interpretation is reductive; facts and events involving the electric field are reduced to facts and events involving distant charged bodies. The vector *E* is merely a mathematical artifact—a bit of formalism representing something about distant charges, not something about an entity existing in addition to distant charges. Hence, *E* can be said to cause a charged body to feel a force only insofar as distant charges cause the force. On this interpretation, electric forces involve action at a distance.

The reductive interpretation deems the electric field to be merely a convenient theoretical device; what makes a remark about the "electric field" true is the sum of certain combinations of various quantities characterizing distant charged bodies. Likewise, in the quoted passage, Russell declares a body's instantaneous velocity to be merely a mathematical artifact; what makes a remark about the body's $v(t_0)$ true is a relation among the body's positions at t_0 and neighboring moments. (Analogous considerations apply to instantaneous acceleration.) The reductive interpretation of *E* depicts the electric field's power to cause forces as requiring that distant charges have the power to cause forces. Likewise, Russell's reductive view must regard a body's instantaneous velocity as bringing something about only if a certain relation's holding among points in the body's trajectory does so.

What does a body's instantaneous velocity bring about? It figures in causal explanations of the body's subsequent trajectory. In accordance with Newton's second law of motion, a body's trajectory in the interval $<t_0, t_0 + \Delta t]$[6] can be causally explained by the body's mass, the forces on the body at each moment in the interval $[t_0, t_0 + \Delta t]$, and some initial conditions: the body's position at t_0 and (here comes our concern) the body's velocity at t_0. (I shall assume that the body feels merely external forces shoving it around—that it cannot break apart, ignite, etc.) But as we have just seen, the body's $v(t_0)$ is a cause, under the reductive interpretation, only if the body's trajectory in a neighborhood of t_0 is a cause—and any such neighborhood includes moments after t_0. Hence, in order for $v(t_0)$ to be a cause of the body's trajectory in $<t_0, t_0 + \Delta t]$, the body's trajectory in $<t_0, t_0 + \Delta t]$ would have to be a cause of itself. This cannot be. Here we have the germ of a powerful argument against the reductive view: that it cannot account for velocity's causal and explanatory role.[7] I shall call this the "causal explanation problem."[8]

I have just suggested that if the electric field here now is a cause of a force on a charged body here now, and if facts and events involving the electric field are just facts and events involving relations holding among the properties of distant charges, then the distant charges' having these properties must be causes of the force. Analogously, if the body's $v(t_0)$ is a cause of the body's trajectory after t_0, and if the body's $v(t_0)$ is just a relation's holding among points in the body's trajectory at t_0 and neighboring moments, then those points in the body's trajectory must be causes of the body's trajectory after t_0. These suggestions presuppose that a relation's holding is a cause only if the relata are. More explicitly:

> If a cause of *e* is a relation's holding among *a*'s possessing property *A*, *b*'s possessing property *B*, etc., then *a*'s possessing property *A* is a cause of *e*, and *b*'s possessing property *B* is a cause of *e*, etc. (where *a* may be identical to *b*).[9]

This presupposition seems plausible. For example, suppose that object *a* occupies the left weighing pan of a balance and object *b* occupies the right. The balance tips toward *a* because of the relation between *a*'s mass and *b*'s: *a*'s is greater. The principle that we presupposed above makes the sensible demand that *a*'s mass and *b*'s mass each have influenced the balance. (Newtonian physics says that they exerted opposite but unequal forces on it.) Otherwise, it couldn't be that the relation between their masses caused the balance to tip. For example, if instead the "balance" is defective so that it cannot tip toward the right side, where *b* is located, then *b*'s mass is not a cause of the balance's tipping, and (as our presupposition requires) the balance tips toward *a* not because *a*'s mass exceeds *b*'s, but rather because *a* (having non-zero mass) occupies the left pan.

Here is another example of our presupposition at work. Suppose a rock's having greater density than the liquid in which it is placed causes the rock to sink. Then by our presupposition, the rock's density (as well as the liquid's) is a cause of its sinking. Moreover, the rock's density is nothing more than a relation's holding between its mass and its volume. By our presupposition, once again, the rock's mass is a cause of its sinking, and so is the rock's volume.[10]

The reductive view interprets $v(t_0)$ as a relation's holding among the points in the body's trajectory at t_0 and neighboring moments. But which neighboring moments are these? In defining $v(t_0)$, we may select an arbitrarily small neighborhood around t_0; for any moment T in $<t_0, t_0 + \Delta t]$, we may select a neighborhood that excludes T. But although no particular point of the body's trajectory (other than $x(t_0)$) is indispensable to the body's having $v(t_0)$, this does not make $v(t_0)$ an intrin-

sic property of the body at t_0. On the reductive view, the body's having $v(t_0)$ depends on the body's locations at instants other than t_0. (The intrinsic-relational terminology is thus potentially misleading when it comes to properties defined in terms of limits, as instantaneous rates of change generally are.) The reductive view portrays $v(t_0)$ as a relation among the points in the body's trajectory in the interval $[t_0 - \Delta t, t_0 + \Delta t]$, and the reductive view also portrays $v(t_0)$ as a relation among the points in the body's trajectory in the subinterval $[t_0 - \delta t, t_0 + \delta t]$, where $\Delta t > \delta t > 0$, but the reductive view does not portray the trajectory's points inside the larger interval but outside the smaller as irrelevant to $v(t_0)$—since then the reductive view would deem every point other than $x(t_0)$ to be irrelevant, even though $v(t_0)$ is not an intrinsic property of the body at t_0.

So on the reductive view, what does it take in order for a baseball's instantaneous velocity upon leaving a pitcher's hand to serve as an initial condition in a causal explanation of the ball's trajectory over the course of the succeeding Δt, during which it travels to home plate? No matter how small a neighborhood surrounding t_0 we select, part of it is after t_0. Select a neighborhood and take a moment T in that part. For $v(t_0)$ to be a cause of the body's $x(T)$, where the body's possessing $v(t_0)$ is a relation's holding among the points in the body's trajectory in that neighborhood, our presupposition says that every relatum must be a cause of the body's $x(T)$. So $x(T)$ must be a cause of itself. (And $x(T')$, where T' is *after* T and belongs to the neighborhood we have selected, must also be a cause of $x(T)$.) This is problematic.

However, $v(t_0)$'s character as a limit suggests that there may be a way for the reductive view to get around the causal explanation problem. The body's trajectory during $<t_0, t_0 + \Delta t]$ is nothing but all of the ordered pairs (x, t) for moments in $< t_0, t_0 + \Delta t]$. For each moment T in $< t_0, t_0 + \Delta t]$, there is some neighborhood around t_0 that excludes T where the body's trajectory over that neighborhood suffices to fix $v(t_0)$. Perhaps, then, $v(t_0)$ can serve as an initial condition in causally explaining the body's trajectory over $<t_0, t_0 + \Delta t]$, even though it is nothing but a relation's holding among points in the body's trajectory, because each point in the body's trajectory in $<t_0, t_0 + \Delta t]$ can be causally explained by other points that suffice to fix $v(t_0)$. By explaining the trajectory in $< t_0, t_0 + \Delta t]$ point by point, no point in that trajectory has to help explain itself. In other words, an explanation of the body's trajectory in $< t_0, t_0 + \Delta t]$, with $v(t_0)$ as an initial condition, could be interpreted as follows: for any point $x(T)$ of that trajectory, there is a causal explanation with one initial condition consisting of a certain relation's holding among the points in the body's trajectory in any neighborhood around t_0 that is small enough to exclude T.

Does this proposal respect the principle we presupposed: that a relation's holding is a cause only if the relata are? Not quite. On this proposal, the body's $v(t_0)$ is a cause of the body's $x(t_0 + \delta t)$ and is a relation's holding among the points in the body's trajectory in $[t_0 - \Delta t, t_0 + \Delta t]$. Yet not all of those relata are causes of the body's $x(t_0 + \delta t)$; only those $x(t)$'s where $t < (t_0 + \delta t)$ are causes. Of course, it might be suggested that when the relation involves a limit, the principle that a relation's holding is a cause only if the relata are should not be required to apply fully, but only to this extent.

However, an important aspect of $v(t_0)$'s causal role in classical physics is that it serves as a cause of *all* of the points in the body's trajectory in $<t_0, t_0 + \Delta t]$. It is a *common* cause of the body's position at every later moment—no matter how remote from t_0, and certainly no matter how near. On the above proposal, the common cause we find in $v(t_0)$, a relation's holding among the points in the body's trajectory, is lost when we proceed to take the relata as causes. For any two points $x(t_0 + \Delta t)$ and $x(t_0 + \delta t)$ in the body's trajectory after t_0 (indeed, for any finite number of points), there is a neighborhood around t_0 where all of the points in the body's trajectory in that neighborhood, fixing $v(t_0)$, can serve as common causes of $x(t_0 + \Delta t)$ and $x(t_0 + \delta t)$. But no single neighborhood can play this common-causal role for *all* of the points in the body's trajectory in $<t_0, t_0 + \Delta t]$. Thus, the reductive view fails to respect a slight extension of the principle we presupposed (that a relation's holding is a cause only if the relata are)—namely, that a relation's holding is a *common* cause only if the relata are.

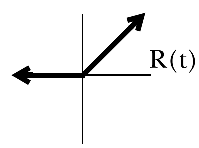
This extension seems as plausible as the original principle. For example, on both occasions when we placed objects *a* and *b* on the balance, it tipped toward *a*. These two outcomes have a common cause: *a*'s mass exceeding *b*'s. This relation's holding is a common cause only because *a*'s mass and *b*'s mass, the relata, are common causes—just as our extended principle requires. Now suppose that there was a further occasion on which we placed object *c* along with *b* on the right pan of the balance, and the balance still tipped to the left—toward object *a*. Is *a*'s mass exceeding the sum of *b*'s and *c*'s masses now a common cause of the balance's having tipped toward *a* on all three occasions? I think not, and our extended principle agrees. In order for that relation's holding among *a*'s, *b*'s, and *c*'s masses to be a common cause, according to our extended principle, those relata would all have to be causes of each outcome. But *c*'s mass was not a cause of the outcome on those occasions when *c* was not on the right pan.

On the reductive view, there is no stretch of the body's trajectory before and after t_0 that can play $v(t_0)$'s role as a cause of each point in the body's trajectory after t_0. It seems hopeless, then, to try to reduce $v(t_0)$ to a feature of the body's trajectory before and after t_0, since whatever $v(t_0)$ is reduced to must be a common cause of every point in the body's trajectory after t_0. However, this problem may be avoided by a slightly different reductive approach, to which I now turn.

2. Velocity as the Trajectory's Derivative from Below?

Consider the ramp function:

$$R(t) = \begin{cases} 0 \text{ for } t \leq 0 \\ t \text{ for } t \geq 0. \end{cases}$$



This function is continuous, but at t = 0, it is nondifferentiable. The graph of R(t) is "bent" at t=0. We can better express R's behavior at t=0 by considering its derivative "from below" and its derivative "from above." For any function f(x), we can define its limit as x approaches a "from below" (a.k.a. "from the left") by amending our earlier definition of "$\lim f(x) = L$": where we had stipulated "whenever $0 < |x - a| < \delta$", we merely add the requirement that $x<a$. That is, whereas we had been taking a neighborhood of x=a that extends for a distance δ above and δ below a, now in taking the limit *from below* we are considering only the neighborhood extending δ below a. Analogously, we may define a function's limit as x approaches a "from above" (a.k.a. "from the right"). It is easily shown that f has a limit *simpliciter* at x=a if and only if its limits there from above and from below exist and are equal. We may use the limits from below and from above to define a function's derivative from below and derivative from above, respectively. As its graph suggests, R's derivative from below at t=0 is zero, whereas its derivative from above at t=0 is one. Since these are unequal, R(t) has no derivative at t=0.

It is tempting to try to avoid the causal explanation problem by tweaking Russell's view and identifying a body's velocity as its trajectory's time-derivative *from below.*[11] Then apparently, the body's $v(t_0)$ can serve as an initial condition in explaining the body's trajectory over the interval $< t_0, t_0 + \Delta t]$. For each moment T in $< t_0, t_0 + \Delta t]$, x(T) has a causal explanation with a relation's holding among the points in the body's trajectory in any neighborhood extending below but not above t_0 as an initial condition. Since each of those neighborhoods extends only *below* t_0, each can serve for all moments in $< t_0, t_0 + \Delta t]$. So we have apparently found a way to reduce $v(t_0)$ to a feature of the body's trajectory while respecting $v(t_0)$'s role as a common cause of all of the points in the body's trajectory in $< t_0, t_0 + \Delta t]$.

To elaborate the causal explanation problem, it will be useful to see that this derivative-from-below proposal, although inadequate, cannot be defeated as easily as some critics of the reductive view may believe.

One objection to this proposal is that it portrays local causation as involving action at a temporal distance. Bigelow and Pargetter offer this argument:

> Consider, for instance, a meteor striking Mars, and consider the problem of explaining why it creates a crater of precisely the size that it does. At the moment of impact, the meteor exerts a specific force on the surface of Mars. Why does it exert precisely that force? Because it is moving at a particular velocity relative to Mars. On the Ockhamist view, it exerts the force it does because it has occupied such-and-such positions at such-and-such times. In other words, the Ockhamist appeals to the positions the meteor has occupied in the past. But why should a body's *past* positions determine any force *now*? This requires the meteor to have a kind of "memory"—what it does to Mars depends not only on its current properties but also on where it has been. (1990, 72)

But there is a complete cause of the meteor's impact force at t_0 that lies entirely within Δt of t_0: the meteor's positions at $[t_0 - \Delta t, t_0]$ fix the meteor's $v(t_0)$, interpreted as the trajectory's time-derivative at t_0 from below. We can make Δt as small as we like (as long as it remains non-zero). This interaction, then, exhibits the sort of temporal locality that, according to Lange 2002a (7–17), suits a universe where time is dense—that is, where we cannot require that the cause occur at the point immediately preceding the effect, since there is no such point. Temporally local causation occurs when there is no finite temporal gap separating an effect from every complete set of its causes. There is no finite, non-zero Δt such that the meteor needs to "remember" events that took place more than Δt ago.[12]

What if the body exists at t_0 but the body's trajectory has no time-derivative from below at t_0 because the body's most recent moment of existence before t_0 was across a temporal gap: at $t_0 - \Delta t$ (for some finite, non-zero Δt)? According to some philosophers, such as Tooley (1988, 247–48), a body pursuing such a "winking" trajectory might nevertheless intuitively have a velocity at t_0, contrary to the derivative-from-below proposal we are considering. However, our concern is not with some alleged ordinary, pretheoretic notion of instantaneous velocity, but rather with the notion that figures in classical physics. A body in classical physics cannot exist at time $t_0 - \Delta t$ and at time t_0 without existing at every moment in between. So the derivative-from-below proposal may be correct in entailing that a body pursuing the winking trajectory possesses no classical instantaneous velocity at t_0.

Suppose that a body's trajectory is given by the ramp function R(t). For instance, suppose that bodies *a* and *b* are ideal, perfectly rigid billiard balls having the same mass and constrained to move in only one dimension. Body *a* is at rest until body *b*, moving with speed 1 unit, collides with body *a* at t=0. After the collision (which is elastic, since the bodies are incompressible), body *b* is at rest and body *a* moves off with speed 1 unit. Body *a*'s trajectory is R(t). What are the bodies' velocities at t=0? Tooley (1988, 246–47) and Carroll (2002, 59–60) suggest that the answer is underdetermined by the bodies' trajectories, contrary both to Russell's reductive view and to the derivative-from-below proposal. In other words, they suggest that it could be that at t=0, *a*'s speed is 1 unit and *b* is at rest, and it also could be that at t=0, *a* is at rest and *b*'s speed is 1 unit.

Russell's view entails that both bodies' instantaneous velocities at t=0 are undefined (since *a*'s trajectory's derivative from above is unequal to its derivative from below, and likewise for *b*). But then the system's total energy at the moment of collision (which is *a*'s kinetic energy plus *b*'s, where a body's kinetic energy is equal to half its mass times its velocity squared) is undefined, making it difficult to see how the system obeys the law of energy conservation. (The same applies to momentum conservation.) The derivative-from-below proposal entails that at t=0, *a* is at rest and *b*'s speed is 1 unit.[13] However, it seems arbitrary to award the motion at t=0 to *b* rather than to *a*. This arbitrariness is apparently part of Tooley's and Carroll's motivation for suggesting that the bodies' instantaneous velocities at t=0 are underdetermined by their trajectories.

Indeed, Tooley and Carroll appear to suggest that the laws of classical physics would be obeyed were *a* at rest and *b* moving at t=0, and those laws would also be obeyed were the reverse the case. On this view, the bodies' instantaneous velocities at t=0 (and the corresponding distribution of energy and momentum between the bodies at t=0) are determinate but make no difference to anything else. However, physics ordinarily regards this sort of ontological dangler as extremely suspicious—as symptomatic of there being no fact of the matter at issue. An attractive explanation of why some theoretical quantity makes no difference to anything else is that it is unreal; it constitutes surplus ontological structure. Consider the electric potential's absolute value, whose dangling character led Faraday and Maxwell to believe it unreal. Or consider a body's absolute velocity, whose dangling character led Einstein to develop relativity theory, according to which there are no absolute velocities. But in interpreting classical physics, we do not want to conclude that there is no fact of the matter about the velocities of bodies *a* and *b* at t=0. (Energy and momentum conservation would then be inapplicable to t=0.)

To deal properly with this example, we must consider not only velocity at t=0, but also acceleration at that moment. If *a* is moving uniformly with speed 1 unit at and after t=0, then *a*'s acceleration at t=0 is infinite if *a*'s acceleration is its velocity's derivative from below, and *a*'s acceleration at every moment (including t=0) is zero if *a*'s acceleration is its velocity's derivative from above. (Analogous results apply to *b*'s acceleration at t=0 if *a* is at rest and *b* is moving uniformly with speed 1 unit at and before t=0.) For that matter, even a conception of acceleration as one of a body's primitive properties (alongside its mass and electric charge) would appear compelled to ascribe to *a* and *b* at every moment either zero or infinite acceleration. But neither of these attributions is plausible as part of a causal interpretation of classical physics, where all changes in velocity are caused by accelerations that, in turn, are all caused by forces. Since the velocities of bodies *a* and *b* change in the collision, the bodies must undergo a non-zero acceleration in the collision, and hence (by $F = ma$) must feel a non-zero net force. But even if we are prepared to countenance a momentary infinite force, a momentary infinite acceleration cannot be plugged into the causal law governing how a body's final velocity $v(t_2)$ at the end of the interval $[t_1, t_2]$ is caused by its initial velocity $v(t_1)$ and the instantaneous acceleration $a(t)$ that it feels at each moment during that interval:

$$v(t_2) = v(t_1) + \int_{t_1}^{t_2} a(t)\, dt$$

If $a(t)$ equals zero at all times except t=0, when it becomes infinite, then $a(t)$ cannot be integrated.[14]

Of course, we do not need to integrate a(t) in order to predict the outcome (after t=0) of *b*'s collision with *a*. The laws of energy and momentum conservation (and the bodies' impenetrability) suffice. In this way, classical physics can successfully predict the outcomes of certain actual collisions by approximating them as ideal, momentary, perfectly elastic collisions. (This is known as the "impulse approximation.") But the conservation laws do not supply a *causal* account of the collision, an account that explains the bodies' final velocities in terms of the forces the bodies feel and the accelerations they undergo. They explain the outcome non-causally.[15]

I conclude that classical physics (interpreted causally) regards *a*'s and *b*'s trajectories as impossible, since they involve momentary, finite changes in velocity, which would require infinite instantaneous accelerations, which cannot be accommodated by the law by which instantaneous acceleration causes changes in velocity. Although the impulse approximation is good enough for many practical purposes when dealing with collisions that occur very quickly, the impulse approximation does not give a causal explanation of a collision's outcome. Collisions that take only a moment to occur do not take place in any possible world operating according to classical physics (interpreted causally). In such a possible world, instantaneous velocity is always differentiable. All classical collisions involve finite forces that vary over the course of finite intervals. Possible worlds operating according to classical physics may incorporate many idealizations as compared to the actual world (such as infinite, perfectly flat, frictionless surfaces). But collisions cannot be idealized as momentary without making it impossible for classical physics to explain them causally.[16] Therefore, in interpreting classical instantaneous velocity, we do not need to find some reason to break the symmetry between *a* and *b* in order to decide which one is moving at t=0, nor must we acquiesce to the unpalatable consequences of deciding that the bodies' instantaneous velocities at t=0 are ontological danglers.[17]

Several philosophers have considered a possible world where a body's sequence of positions is determined at random or by God's will (as in occasionalism). Trajectories in this world tend to be highly discontinuous, but suppose that a body accidentally (or by God's will) moves along a differentiable path over the interval $< t_0 - \Delta t \, , \, t_0 + \Delta t >$. Intuitively, according to Bigelow and Pargetter (1990, 68–70) and Tooley (1988, 244), the body has no $v(t_0)$—or, according to Carroll (2002, 56), it may perhaps have no $v(t_0)$—although it definitely does have a velocity on Russell's reductive account and on the derivative-from-below proposal that we are now examining. The body allegedly has no $v(t_0)$ because velocity's explanatory role in classical physics is essential to it, but in this weird possible world, the body's "velocity" at t_0, as the reductive account (or the derivative-from-below proposal) defines it, fails to help explain the body's subsequent trajectory. On the other hand, advocates of the reductive account or the derivative-from-below proposal could apparently bite the bullet, as Meyer (2003, 99) does, and say that the body *has* a well-defined $v(t_0)$, insisting that velocity's explanatory role in a world operating according to classical physics is beholden to contingent laws of nature that are not laws in this weird possible world. The traditional view, of course, is that all properties owe their explanatory roles to contingent laws of nature; a scientific explanation cannot get by on initial conditions, logical truths, and metaphysical necessities alone, but also requires natural laws. So as yet, we have no decisive argument against the derivative-from-below proposal.

What about a body whose trajectory has no time-derivative from below at t_0 because the body first comes into existence at t_0? This trajectory (at last!) seems perfectly admissible within classical physics. Suppose that at t_0, a bullet penetrates a wooden block at rest on a frictionless plane. Within the block, the bullet slows dramatically; it becomes lodged in the block. Thus, when the bullet enters the block at t_0, a new combined body is created. Consider the ideal case where none of the bullet's initial energy goes toward heating the block. The newly created body's instantaneous velocity v, at some time after the bullet comes to rest relative to the block, is given by momentum conservation:

$$v = m_b v_b / (m_b + m_w),$$

where m_b is the bullet's mass, m_w is the wooden block's mass, and v_b is the bullet's pre-collision velocity. But on the derivative-from-below proposal, the new body's instantaneous velocity at t_0 is not well defined; its trajectory has no time-derivative from below at t_0, since t_0 was this body's first moment of existence.

A solution to this problem is readily available.[18] The "newly created" body is just an amalgamation of two pre-existing bodies. This point generalizes in classical physics. The law of mass conservation says that in a closed system, the total mass remains constant. Some can disappear here if an equal quantity appears there. But classical physics enshrines not merely the *conservation*, but also the *continuity* of mass: for a certain region of space to contain more mass at t_2 than at t_1, some mass during the interval must have flowed into the region across the surface enclosing it.[19] In short, when a body is "created," its mass must have come from somewhere. These somewheres can supply our missing derivatives from below.

The new block-and-bullet body's mass obviously derives from the pre-collision bullet and block. The mass parcel deriving from the bullet has a trajectory with time-derivative from below at t_0 equalling v_b. The mass parcel deriving from the block has a trajectory with time-derivative from below at t_0 equalling zero. The average of these, with contributions weighted according to their masses, is exactly v as given above by momentum conservation. This is the "new" body's velocity at t_0. Thus, velocity as the trajectory's time-derivative from below can accommodate "newly created" bodies.[20]

So far, then, by defining $v(t_0)$ and $a(t_0)$ as the trajectory's first and second time-derivatives (respectively) at t_0 from below, we have apparently allowed these quantities to play their traditional roles as causes. But what if they are serving as *effects* and not (merely) as causes? Interpreting $v(t_0)$ and $a(t_0)$ as relations holding among the points in the body's trajectory at and before t_0 may be advantageous when $v(t_0)$ and $a(t_0)$ are serving as causes, but it is inconvenient when they are serving as effects.

For example, watch what happens when the body's instantaneous acceleration $a(t_0)$ is an effect. Suppose that a charged body is located within an electric field (and feels no other forces). Then the body's $a(t_0)$ is caused by its mass and the electric force it feels (in accordance with $F = ma$). That force $F(t_0)$, in turn, is caused (in accordance with $F = qE$) by the field E at a given location, along with the body's possessing charge q and occupying that location $x(t_0)$ at t_0. So $x(t_0)$ is a cause of $a(t_0)$. But according to the derivative-from-below proposal, $a(t_0)$ is a relation's holding among the points in the body's trajectory in a neighborhood from some earlier moment up to and including t_0. I suggested earlier that a relation's holding is a cause only if each of the relata are. I now suggest that analogously, a relation's holding is an effect only if at least one relatum is. (If no relatum is affected, then how can their relationship have been affected? For example, suppose that a cause of my current weight's being below yours is my having recently followed a strict diet. Then my dieting must have affected either my current weight or yours.)[21] So for $x(t_0)$ to be a cause of $a(t_0)$, as instantaneous acceleration is understood by the derivative-from-below proposal, $x(t_0)$

must be a cause of some point in the body's trajectory in an interval ending at t_0. But $x(t_0)$ cannot cause itself, and the other trajectory points in that interval occur *before* $x(t_0)$, so it cannot cause them.[22]

This problem arose from considering the body's instantaneous state of motion as an effect, whereas we had originally been concentrating on its role as a cause of the body's subsequent trajectory. Of course, we might try to avoid this problem by positing two distinct instantaneous velocities and two distinct instantaneous accelerations, one property of each pair involving the trajectory's time-derivative at t_0 from below, and the other involving its derivative from above. The belows are the causes; the aboves are the effects. For example, the trajectory's first derivative from below at t_0 is the $v(t_0)$ that serves as an initial condition—a cause of the body's trajectory in a subsequent interval $<t_0, t_0 + \Delta t]$. The trajectory's second derivative from above at t_0 is the $a(t_0)$ that serves as an effect of the body's $x(t_0)$, the electric field there then, and the body's mass and charge. On this view, talk of "$v(t_0)$" (or "$a(t_0)$") is ambiguous. Several philosophers, such as Jackson and Pargetter (1988) and Meyer (2003, 97), have offered proposals along roughly these lines.

One problem with the two-property view involves deciding which of the two velocity properties should figure in other properties to which velocity is tied, such as kinetic energy ($\frac{1}{2}mv^2$) and momentum (mv). Perhaps these properties come in pairs as well, necessitating at least a doubling of all properties in which these, in turn, figure. This is fairly odd though, perhaps, we could bear it.

A more serious problem with the two-property view is that if $v(t_0)$ as effect is distinct from (even if sometimes equal to) $v(t_0)$ as cause, then there exists no single causal chain running *through* $v(t_0)$—that is, with the same $v(t_0)$ as effect *and* as cause. It cannot be the case, for example, that the body's trajectory in $[t_0 - \Delta t, t_0>$ helped to cause its $v(t_0)$ which, in turn, helped to cause its trajectory in $<t_0, t_0 + \Delta t]$. The same applies to acceleration. If there are two acceleration properties, then it is not the case that the force on a charged body causes its acceleration, which, in turn, affects its electric field (Feynman et al. 1963, 2:21-1). Rather, no common node joins the two halves of this causal chain. This seems like a high price to pay.

3. Velocity's Essentially Kinematic Character

According to velocity primitivism, a body's velocity is a property over and above its trajectory. The two are metaphysically independent, though connected by natural laws that give velocity its explanatory and causal roles. A common complaint against primitivism—lodged, for example, by Arntzenius (2000, 197)—is that it fails to respect ontological parsimony. That is, it adds to our ontology a property that we can dispense with (having already admitted trajectories) as well as a natural law reflecting this new property's redundancy, since this law merely sets the new quantity equal to the trajectory's time-derivative (at least when things are well behaved).

Of course, the contrary is suggested by the causal explanation problem. The additional property seems absolutely indispensable, since velocity—as the reductive view construes it—cannot stand in the requisite causal and explanatory relations. Nevertheless, I think that there is something fundamentally mistaken about primitivism: it fails to do justice to the fact that velocity is *essentially* something to do with trajectory (and nothing more than that).

Suppose that velocity were a property on an ontological par with position. What property would it be? We would understand velocity's nomic, causal, and explanatory roles, but not what it *is* for a body to have an instantaneous speed of 5 centimeters per second. We would know what velocity does but not what velocity is intrinsically.

Perhaps exactly this is the case for certain fundamental properties, such as a body's inertial mass, magnetic moment, and electric charge, or an electric field's strength and direction. Perhaps we do not know what it is in virtue of which a body possesses a charge or what it is that makes a positively charged body different from a negatively charged body. Perhaps we know only what difference this difference makes to the body's behavior under various circumstances.[23]

Although this may not actually be the right thing to say about such primitive properties as mass and electric charge, I recognize the attraction of this view.[24] But I am not at all tempted to regard velocity in a similar way. This suggests that velocity primitivism is incorrect. The idea that what it is to be moving at 5 centimeters per second is something-we-know-not-what is as bizarre as the idea that what it is in itself to be to the left of a given object is something-we-know-not-what—or what it is to be 5 centimeters long (or 5 seconds in duration) is something-we-know-not-what. Unlike mass and electric charge, velocity is an essentially kinematic property: it is no more ineffable than trajectory because whatever velocity is, we know that it must have something to do with trajectory (even if a body's $v(t_0)$ fails to supervene on its trajectory). That is, we know that trajectory, velocity, and acceleration belong to the same "family" of properties; velocity's place in this family is essential to it. (I will shortly be more explicit about what this means.) So there is no temptation to say that we know nothing about what velocity is, only about what velocity does. The same considerations apply to other rates of change. Darkening cannot be something ineffable over and above anything to do with degrees of darkness; cooling cannot be something metaphysically independent from temperature.

The same thought can be motivated by the traditional view of natural laws as contingent truths.[25] On that view, it is metaphysically possible for like electric charges to attract rather than to repel. It is metaphysically possible for electric charge to have played mass's nomic role, and vice versa. But it is far more difficult to see how mass could have stood in speed's relation to trajectory, or how velocity could have played acceleration's role and vice versa, or how magnetic moment could have played velocity's role. Having magnetic moment and velocity interchange their roles would be as bizarre as having mass and position do so. Velocity seems no more "interchangeable" than position, unlike (say) the various quark "colors," which seem utterly interchangeable. (All that seems to matter is their distinctness from one another, not what they are in themselves.)

It follows that certain relations among trajectory, velocity, and acceleration are metaphysical necessities rather than natural laws, as velocity primitivism takes them to be.[26] For example, it is a logical truth that if a body has well-defined position, velocity, and acceleration throughout the interval $[t_0, t_0 + \Delta t]$, then

$$v(t_0+\Delta t) = v(t_0) + \int_{t_0}^{t_0+\Delta t} a(t)\, dt$$

and

$$x(t_0+\Delta t) = x(t_0) + \int_{t_0}^{t_0+\Delta t} v(t)\, dt\,.$$

These relations express the way that acceleration, velocity, and trajectory essentially belong to the same "family" of properties. Within this family, acceleration's relation to velocity is just like velocity's relation to position. These relations (along with Newton's second law of motion) are used to explain a body's trajectory during $<t_0, t_0 + \Delta t]$ by appealing to the net force on the body during $[t_0, t_0 + \Delta t]$, the body's mass, and initial conditions $x(t_0)$ and $v(t_0)$. This is the scientific explanation that originally prompted the causal explanation problem.

We might worry that if these relations are metaphysically necessary, but every scientific explanation requires contingent laws of nature, then the scientific explanations that prompted the causal explanation problem cannot be genuine scientific explanations. However, for $v(t_0)$ to help explain the body's subsequent trajectory, the body must *have* a subsequent trajectory; it must continue to exist. (Notice that one of the above metaphysical necessities is that $v(t_0)$ stands in a certain relation to the body's acceleration over the course of a subsequent interval, and to its velocity at that interval's end, *if* the body has well-defined position, velocity, and acceleration throughout the interval. So this metaphysical necessity does nothing to preclude a body from having a well-defined $v(t_0)$ while lacking a subsequent trajectory.) Given the initial conditions, a contingent law of nature is needed to ensure that the body will continue to exist. This part of the explanation is implicit in using a body's velocity during $[t_0, t_0 + \Delta t]$, its $x(t_0)$, and the above relation between position and velocity to explain a body's $x(t_0 + \Delta t)$ since, in appealing to the body's velocity during $[t_0, t_0 + \Delta t]$, we imply that the body exists throughout this interval. Analogous considerations apply to any scientific explanation in which a quantity's value at t_0 and its instantaneous rate of change in $[t_0, t_0 + \Delta t]$ explains its value at $(t_0 + \Delta t)$. (I shall say more about this in the next section.)

The idea that velocity is essentially kinematic is distinct from ontological parsimony and, I believe, as strong a motivation behind the reductive view. Let's now see how to respect velocity's essentially kinematic character without adopting the reductive view. The resulting account will thereby solve the causal explanation problem.

4. Velocity as Like a Disposition

I propose that classical instantaneous velocity is something like a dispositional property, a tendency, a power, or a propensity. To be moving at t_0 in a given direction at 5 centimeters per second is to have a certain *potential* trajectory—that is, to be *threatening* to proceed along a certain trajectory. It is to be disposed at t_0 to have a trajectory with a time-derivative *from above* at t_0 equal to 5 cm/s in the given direction.

A disposition need not be manifested, of course. A body's fragility, for instance, need not manifest itself; a fragile object may never break, since it may never be subjected to rough treatment. It merely would break, were it treated roughly (under the appropriate circumstances).[27] The requisite condition for $v(t_0)$'s manifesting itself is just that the body exist after t_0. Here, then, is my proposal:

> What it is for a body to have $v(t_0) = 5$ cm/s is for the body to exist at t_0 and for it to be true at t_0 that were the body to exist after t_0, the body's trajectory would have a time-derivative from above at t_0 equal to 5 cm/s.

The time-derivative is taken *from above* in accordance with the familiar stimulus-response construal of dispositions: a body at t_0 has a certain disposition exactly when it is disposed at t_0 to respond in a certain way to a certain stimulus under certain conditions. The response does not occur prior to the stimulus, since the stimulus helps to cause the response. In the case of $v(t_0)$, the body's continuing to exist after t_0 is not exactly a *stimulus*—a *cause* of the response (the body's subsequent trajectory's having a certain character). Rather, the body's continuing to exist after t_0 indicates the *absence* of any factor at t_0 that would keep the body from fulfilling its potential (as far as its trajectory is concerned) by causing it to cease to exist after t_0.

I propose that acceleration be understood in nearly a similar way: $a(t_0)$ involves a subjunctive fact concerning what $x(t)$'s second time-derivative from above at t_0 would be, were the body to continue to exist for some finite temporal interval after t_0 *and* to experience then exactly the same net force as it actually experiences at t_0. This last condition is required to accommodate the fact that in classical physics, a force field's non-zero region may have a sharp boundary—as when the electric field is zero inside and non-zero at and outside of a hollow, charged, conducting spherical shell. Consider a charged body on the shell but heading inside of it. The body has non-zero $a(t_0)$ but, were it to continue to exist, its trajectory's second time-derivative from above at t_0 would (neglecting non-electrical influences) be zero, since it would thenceforth be in a region of zero electric field. However, were it to continue to exist *while experiencing the same net force as it actually experiences at t_0*, its trajectory's second time-derivative from above at t_0

would be non-zero. No similar condition needs to be added to the definition of velocity because in classical physics, a discontinuity in a field produces a discontinuity in acceleration, not in velocity.[28] In a possible world operating according to classical physics (interpreted causally), outside influences act directly not upon a body's trajectory or upon its instantaneous velocity, but rather upon its velocity's rate of change. This suggests an important point (to which I shall return) regarding change in general: some quantities change because something sets them at a new value (like a thermostat being reset), whereas other quantities change because something elevates their rate of change (or the rate of change of their rate of change, or …) to a non-zero value. A rate of change (such as acceleration) that is directly set by outside influences must be understood in terms of a slightly different sort of subjunctive conditional than a rate of change (such as velocity) that changes exclusively because outside factors directly set *its* rate of change.

Considering that my aim is to give an analysis of classical instantaneous velocity that will function adequately in a traditional causal interpretation of classical physics, we should not be surprised that analyses roughly like mine were offered long ago (though since the late nineteenth century, they have been neglected in favor of the reductive view). Thomson and Tait "define the exact velocity [of a point body] as the space which the point would have described in one second, if for one second its velocity remained unchanged" (1888, 12). A similar account is offered by Maclaurin, who calls instantaneous velocity a "power" (Maclaurin 1742, 53–55; see Jesseph 1993, 281–82, and Carroll 2002, 66); Walton calls a body's instantaneous velocity the "[t]endency forward in the body" (1735, 47). Maclaurin says that "the velocity of motion is always measured by the space that would be described by that motion continued uniformly for a given time" (1748/1971, 104). (A similar definition is given by Hutton 1796, 484.) Although these proposals are similar to mine in having a subjunctive character, they appear to be circular; in defining a body's instantaneous velocity at t_0, these proposals appeal to the body's instantaneous velocity's remaining constant over a finite period beginning at t_0. These authors might have appealed to zero instantaneous acceleration instead of unchanged instantaneous velocity. However, they would then have had to offer a non-circular analysis of instantaneous acceleration (which they fail to do). These problems do not arise on my proposal, since it defines $v(t_0)$ in terms of what the body's trajectory after t_0 would be like, were the body to exist after t_0—without specifying that the body must maintain a constant velocity at and after t_0. This condition is unnecessary because velocity is differentiable in classical physics, interpreted causally (since velocity is not set directly, but only by forces setting its rate of change, so that rate must be well defined), and because I am defining $v(t_0)$ in terms of the derivative-from-above, not in terms of what the body's path over the next second would have been.[29]

There is a further problem with defining a body's instantaneous velocity in terms of the distance that the body would cover over the next second, were the body to undergo zero instantaneous acceleration throughout that interval. Consider a baseball that has been hit and at t_0 is falling freely; it is undergoing non-zero downward acceleration. Suppose that the outfielder comes close to catching the ball at about t_0, but narrowly misses it. The easiest way for the ball's downward acceleration at t_0 to have been zero is for the outfielder to have caught the ball, in which case the ball's fall would have slowed dramatically once the ball made contact with the outfielder's glove; the ball's downward acceleration at and after t_0 would then have been zero because the ball would by then have been firmly within the fielder's glove, its instantaneous downward velocity zero. So the vertical distance that the ball at t_0 would have covered in the next second, if its downward acceleration during that second had been zero, is zero. Yet that is not the ball's actual downward $v(t_0)$. In short, were the ball's downward acceleration at t_0 to have been zero, its $v(t_0)$ would have been different from what it actually was. My proposal's "were the ball to exist after t_0" sometimes invokes a closer possible world than "were the ball's instantaneous acceleration during the next second to be zero." Of course, it would again be circular to define the ball's $v(t_0)$ in terms of the ball's motion were the ball's instantaneous acceleration from t_0 onward to be zero while its instantaneous $v(t_0)$ remained as it actually was (which would not be the case if the zero acceleration were achieved by the ball's being caught).

On my proposal, a body that exists at t_0 and subsequently, but not before, can have a well-defined $v(t_0)$ despite its trajectory's having no time-derivative (from below or simpliciter) at t_0, since it may nevertheless be disposed to have a certain kind of trajectory after t_0. Likewise, a body that exists at t_0 and before, but not subsequently, can have a well-defined $v(t_0)$ despite its trajectory's having no time-derivative from above (or simpliciter) at t_0. The proper conditions for manifesting its velocity were simply not realized. Had the body with $v(t_0) = 5$ cm/s not gone out of existence at t_0, the body would have pursued a trajectory with time-derivative from above at t_0 equal to 5 cm/s.

On my proposal, certain bodies in certain possible worlds lack classical instantaneous velocities at certain times. (These worlds are not operating according to classical physics (interpreted causally), since classical physics (so interpreted) presupposes that each body, whenever it exists, possesses a well-defined classical instantaneous velocity and acceleration.) For example, suppose that in a given possible world, the laws of nature require bodies to pursue "winking" trajectories: for any time t, a body that exists at t cannot exist at any other moment within (say) 3 seconds of t. Then a body's trajectory cannot have a time-derivative from above at any time. So on my account, no body ever has a classical instantaneous velocity.

Likewise, reconsider the possible world where a body's sequence of positions is determined at random. A body in this world might accidentally move along a differentiable path over the interval $< t_0 - \Delta t\ ,\ t_0 + \Delta t>$. However, on my proposal, even such a body lacks a well-defined $v(t_0)$, since it is not true at t_0 that were the body to exist after t_0, its trajectory at t_0 would have a well-defined time-derivative from above.[30] The body at t_0 is, as it were, awaiting the toss of a cosmic die.

To see this, compare the body in this hypothetical world to an actual object governed by probabilistic laws, such as a radioactive atom. An atom of Polonium-209 has a half-life of 102 years. Suppose it has two possible decay modes; upon decaying, it has a 70% chance of undergoing alpha decay and a 30% chance of instead undergoing beta decay. Consider a given atom of Polonium 209 today. Intuitively, it is false today that were the atom to decay sometime within the next 102 years, it would undergo beta decay, and it is also false today that the atom would undergo alpha decay, were it to decay within the next 102 years. (This is so even if the atom actually proceeds to undergo alpha decay sometime within the next 102 years.)

More generally, the forward-directed subjunctive conditional $p > q$ (where p is logically consistent with the natural laws) is true at time t_0 exactly when, if p is added to a certain (context-sensitive) subset of the facts concerning the universe's history during the period up to and including t_0, then q follows logically from these facts together with the natural laws.[31] In the radioactive decay example ("Were the atom of Polonium-209 to decay in the next 102 years, it would undergo beta decay"), we can add the antecedent to *all* of the facts concerning the universe's history until now, and that result together with the laws fails to entail the consequent, since the relevant laws are probabilistic. So the radioactive-decay subjunctive conditional is false.[32] The same applies at t_0 to "Were the body to continue to exist after t_0, its trajectory at t_0 would have a well-defined time-derivative from above" in the possible world where a body's positions are determined at random. Since this subjunctive conditional is false at t_0, the body has no velocity at t_0.[33]

In a possible world operating according to classical physics (interpreted causally), forces influence a body's trajectory by setting the body's instantaneous acceleration (via $F = ma$). The body's instantaneous acceleration, in turn, affects the body's trajectory by affecting the body's instantaneous velocity. In contrast, in a possible world where the body's sequence of positions is determined at random, the outcomes of cosmic die tosses (as it were) set the trajectory *directly*, not by affecting the body's instantaneous acceleration. In that world, the body's trajectory is not caused by its instantaneous acceleration or velocity; indeed, as we have seen, the body has no instantaneous acceleration or velocity, since the requisite subjunctive facts are absent. This contrast suggests an important distinction between two ways in which quantities can be caused to change. Some quantities, such as a body's $a(t_0)$ in a classical-physics world and a body's $x(t_0)$ in the random-trajectory world, are subject to influences that set them directly to a new value irrespective of their former value. Other quantities change in the manner of velocity and position in the classical-physics world in that the quantity's new value depends on its former value; outside influences act directly only on the quantity's rate of change, or on the rate of change of its rate of change, or on some higher-order rate. A body's temperature at t_1, for example, results from its temperature at t_0 and its instantaneous rate of temperature change at each moment during $[t_0, t_1]$, where a cause of that rate at any moment is the instantaneous rate at which heat energy is then entering or leaving the body, which in turn is caused by the difference between the body's temperature and the temperature of its surroundings. The incoming heat energy does not set the body's new temperature directly, but merely causes its temperature to rise.

If a quantity changes in the former fashion, then that quantity may have no instantaneous rate of change (the analogue of instantaneous velocity). But a quantity must have an instantaneous rate of change if the quantity changes in the latter fashion; the requisite subjunctive facts must exist. Indeed, for any quantity that changes in the latter fashion, the causal explanation problem arises. For example, if the body's instantaneous rate of temperature change at t_0 is a cause of the subsequent "trajectory" taken by the body's temperature, then as we saw in section 1, that rate cannot be a relation's holding among the trajectory's points in a neighborhood surrounding t_0. Moreover, the body's instantaneous rate of temperature change at t_0 is an effect, not merely a cause. If the temperature difference at t_0 between the body and its surroundings is a cause of the body's instantaneous rate of temperature change at t_0, then as we saw in section 2, that rate cannot be the temperature trajectory's time-derivative from below. In this way, the very same issues that I have explored in connection with classical instantaneous velocity arise as well in connection with many other instantaneous rates of change, such as temperature's.

How, then, does my proposal resolve the causal explanation problem? As shown by the possible world where a body's sequence of positions is determined at random, the body's actual trajectory after t_0 is not what makes it true (or false) at t_0 that the body would pursue a certain trajectory after t_0, were the body to exist after t_0. On my proposal, a body's $v(t_0)$ is not a relation's holding among points in the body's actual trajectory at t_0 and neighboring moments. So when the body's $v(t_0)$ serves as an initial condition in causally explaining each point in the body's subsequent trajectory, there is no danger of any of those points being a cause of itself and there is no obstacle to $v(t_0)$ serving as a common cause of each of these points.

On a familiar Humean picture, a body's state at one moment is logically independent of the states of things during the rest of the universe's history. The Humean picture gets its grip on us when, roughly speaking, we imagine placing a momentary time-slice of a body, with its momentary state, into any unoccupied spacetime location in any logically possible universe that does not already contain that body, and inserting into the resulting universe at other unoccupied spacetime locations further momentary time-slices of the body in arbitrary momentary states—and never find ourselves producing a contradiction. But if $v(t_0)$ is part of a body's state at t_0, then on my account of classical instantaneous velocity, a body's state at one moment imposes a logical constraint on what its states can be during the rest of the universe's history: the body cannot continue to exist after t_0 without having a trajectory whose time-derivative from above at t_0 is equal to $v(t_0)$. However, as we have just seen, a body that exists at t_0 is not logically compelled to have a $v(t_0)$ at all; the natural laws must be such that there are at t_0 certain subjunctive facts regarding the body. Any physical quantity's instantaneous rate of change is likewise well defined only by the grace of the natural laws. So if a body's momentary "state" includes its instantaneous velocity, then the body's state imposes logical constraints on the natural laws and thereby on the body's states at other times. On the other hand, if a body's "state" can include only properties (such as mass, position, and electric charge) that are not essentially instantaneous rates of change of other properties, then its state at one moment could indeed be logically independent of its states during the rest of its existence. This seems sufficient to gratify the motivation behind the Humean picture: when we imagine placing momentary time-slices of the body into vacant spots in a logically possible universe, we imagine each time-slice coming with a momentary state involving only properties that are not instantaneous rates of change of other properties.

I have referred to a body's velocity as *something like* a disposition in order to sketch my proposal and to emphasize the role that a subjunctive fact plays in it. However, I resist calling velocity a disposition. Although the metaphysics of dispositions is controversial, the traditional view is that every disposition has a categorical "ground" and it is the ground, rather than the disposition, that helps to cause the disposition's manifestation. For example, the glass's shattering is caused by the ball's colliding with it and the glass's molecular structure, not by its fragility. If the glass's fragility (together with the ball's colliding with it) caused the shattering, then the glass's molecular structure would apparently have no causal work to do, and no laws of nature need be involved in explaining the shattering. To explain the glass's shattering by appealing to its fragility (that it would shatter, were it treated roughly under certain conditions) and its receiving rough treatment under those conditions would be to give an explanation of the dormitive-virtue variety, which is no genuine explanation at all.

However, analogous considerations do not apply to velocity. For velocity (as I have characterized it) to have a "categorical basis" (such as the body's mass and momentum might be thought to be) would conflict with velocity's essentially kinematic character; velocity would involve something over and above its metaphysically necessary connection with trajectory. Admittedly, the relation between a body's $v(t_0)$, its still existing after t_0, and its trajectory's time-derivative from above at t_0 is metaphysically rather than nomically necessary, and so in this respect resembles the relation between an object's fragility, its being treated roughly under certain conditions, and its breaking. However, this resemblance does not keep the body's $v(t_0)$ from being a cause of the body's subsequent trajectory. Whereas the object that breaks is treated roughly at the same moment at which it is fragile, the body's still existing after t_0 concerns the situation *after* the body is characterized by $v(t_0)$. When the body's trajectory's time-derivative from above at t_0 is explained by its $v(t_0)$ and other initial conditions, those conditions are all present at t_0, and so they do not include the body's still existing after t_0. Rather (as I mentioned in section 3), they include the facts that combine with natural laws to explain why the body does not go out of existence at t_0. So when the trajectory's time-derivative from above at t_0 is explained by $v(t_0)$ and other initial conditions, the explanation goes through natural laws and not solely through metaphysical necessities, unlike the pseudo-explanation of the object's breaking because it was fragile and treated roughly. Unlike fragility, then, velocity requires no categorical ground to perform the explanatory duties in its stead. As I shall explain further, I see a fact about a body's instantaneous velocity as a subjunctive fact having no categorical ground.

For a variety of reasons, it may appear unattractive to analyze velocity in terms of a subjunctive fact. One great advantage of the reductive view is that it renders the truth-conditions of "$v(t_0) = 5$ cm/s" unmysterious. A subjunctive conditional's truth-makers have often been thought extremely mysterious indeed. Furthermore, if a body is moving now at 5 cm/s not in virtue of its current categorical properties, but in virtue of where it would be were it to continue to exist, then two bodies could be thoroughly alike now, as far as their categorical properties are concerned, and yet possess different velocities now. The meteor discussed by Bigelow and Pargetter produces a given force upon colliding with Mars not solely because of its current categorical properties and the natural laws, I am contending, but also because of the truth of certain subjunctive conditionals. But that can't be right, Bigelow and Pargetter seem to be saying; these subjunctive conditionals must themselves be made true by categorical facts!

However, it may be somewhat disingenuous to complain that we had hoped to do without subjunctive conditionals in explanations of a body's trajectory. Traditionally, scientific explanations and causal relations have been thought somehow to involve laws of nature, which are distinguished from accidents by the truth of various subjunctive conditionals. Perhaps the truth of certain subjunctive conditionals gives the laws their nomic status (rather than the laws' nomic status making certain subjunctive conditionals true).[34] In that case, insofar as laws explain and help make causal relations hold, the truth of various subjunctive conditionals does.

For that matter, we might question the thought that any subjunctive conditional's truth must be grounded entirely upon the instantiation of various categorical properties. Arguments that have been made against Humean supervenience suggest that two possible worlds, utterly alike in which categorical properties are instantiated at which moments, can differ in which counterfactual conditionals are true there (and in what the natural laws are there).[35] Furthermore, even when the truth of a subjunctive conditional (such as "Were the body treated roughly, it would break") is "grounded" upon various categorical facts (such as the body's molecular structure), those facts fix the subjunctive conditional's truth only by way of certain natural laws. But perhaps, as I just speculated, the laws themselves acquire their nomic status by virtue of the truth of various subjunctive conditionals. So subjunctive facts may be ineliminable. Perhaps the right lesson to be drawn, at long last, from Goodman's (1983, 9–17) famous discussion of cotenability is that there are no wholly categorical truth-conditions for counterfactuals.

Lewis once urged us to reject false preconceptions about what it would be like to understand counterfactuals, and instead to take them "at face value: as statements about possible alternatives to the actual situation" (1986, 161). I endorse the spirit of this remark, but I suggest that we take them as more like statements about the actual situation, just as we take statements about a body's instantaneous velocity to be. While we are accustomed to finding counterfactuals lying in wait behind causal relations, or natural laws, or dispositions, it may be disconcerting to find them bound up within a property as apparently metaphysically innocuous as instantaneous velocity. But Zeno was surely right in finding instantaneous velocity to be far from metaphysically innocuous.

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Notes

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[1] For some examples of this traditional interpretation of classical physics, see Walton 1735, 47; Emerson 1768, v and xi; Maclaurin 1748/1971, 54 and 113–14; and Thomson and Tait 1888, 242 and 385. Philosophers, too, generally presume something like this traditional causal interpretation, as when Lewis takes a car crash as having among its causes the position and velocity of the car a split second before the impact (Lewis 1986, 216; cf. Hempel 1965,

184).

[2] Kuhn 1977 gives a different argument for a similar conclusion.

[3] In *De Caelo* (2.8.290a 25–30), Aristotle says (as far as I can tell) that the moon's always presenting the same face to the earth shows that the moon does not rotate, but merely participates in the rotation of the sphere in which it is embedded. This was also Copernicus's view. (If the moon is embedded in an epicycle whose center revolves around the earth along a deferent, then lunar non-rotation amounts to the moon's keeping the same face toward the center of the epicycle. The moon would then have to rotate in order to keep the same face toward earth.)

[4] I shall be sloppy about vector notation, allowing context to indicate whether I mean speed (a scalar) or velocity (a vector: speed and direction) by ‘v', and likewise for other symbols.

[5] For further discussion, see Lange 2002a.

[6] Following standard notation, a square bracket (“[“ or “]”) indicates that the interval includes its endpoint, and a pointy bracket (“<” or “>”) indicates that the interval omits its endpoint.

[7] Tooley (1988, 240 and 243), Bigelow and Pargetter (1990, 66), and Arntzenius (2000, 192) briefly sketch arguments in a roughly similar spirit. Walton (1735, 47) and Emerson (1768, v and xi) are careful to distinguish instantaneous velocity from its effect (involving change of place).

[8] It might be replied: Yes, instantaneous velocity has no causal role in classical physics. A body's *momentum* does the causal work. By physical law, a body's momentum equals its mass times velocity, but momentum is ontologically distinct from mass and velocity; momentum is ontologically on a par with charge and trajectory. In accordance with Newton's second law of motion (equating the net force on the body at t to the time rate of change at t of the body's momentum), a body's trajectory in the interval $<t_0, t_0 + \Delta t]$ can be causally explained by the body's mass, the forces on the body at each moment in the interval $[t_0, t_0 + \Delta t]$, and some initial conditions: the body's position at t_0 and the body's momentum at t_0. I reply: The causal explanation problem is now reproduced as a puzzle about momentum and its instantaneous rate of change. That rate at t_0 (caused by the net force on the body at t_0) is supposed to be a cause of the body's momentum in $<t_0, t_0 + \Delta t]$. How, then, can momentum's instantaneous rate of change at t_0 be reduced in Russellian fashion to a certain relation among the body's momenta at the instants in a neighborhood of t_0?

It might also be worth considering whether the causal explanation problem disappears in Hamiltonian dynamics. To begin with, it is difficult to interpret Hamiltonian dynamics causally at all, for at least two reasons: (i) in Hamiltonian dynamics, a body's motion is not understood in terms of outside forces impinging upon the body, and (ii) a system's Hamiltonian lives in configuration space rather than in physical space: any generalized coordinates may be chosen, so the resulting equations of motion need not track causal relations. Furthermore, suppose the body's actual trajectory after t_0 is to be explained by that trajectory's possessing the minimum total action of all possible paths to a given point in configuration space beginning from the body's initial condition, which includes its instantaneous $v(t_0)$. Suppose also that $v(t_0)$ is reduced in Russellian fashion to a certain relation's holding among the points in the body's trajectory in a neighborhood of t_0. Then the body's actual trajectory after t_0 must be a cause of itself. (Thanks to Ori Belkind for this last point.)

[9] I intend this principle to be indifferent to whether *e* is a fact or an event, and likewise for the other causal relata. I also leave it open whether *A*, *B*, etc. are intrinsic or relational properties.

[10] The presupposition that a relation's holding is a cause only if the relata are acknowledges relationships as able to be causes. Some philosophers contend that only intrinsic properties (perhaps along with spatiotemporal relations) can be causally relevant. Sometimes this appears as the view that only events can be causes and that events are predominantly intrinsic. (See, for example, Lewis 1986, 262.) I want my argument against Russell's reductive account of instantaneous velocity to remain independent of any such controversial premises. However, those who are willing to embrace such premises could put the causal explanation problem in this way: On the reductive view, a body's $v(t_0)$ is not an intrinsic property of the body at t_0. So a body's having a given instantaneous velocity at t_0 is not an event. It therefore cannot be a cause of the body's subsequent trajectory.

[11] Tooley (1988, 243) considers and rejects this alternative, though he seems to think that it succeeds at least in avoiding the causal explanation problem.

[12] I shall consider a different interpretation of Bigelow and Pargetter's argument near the close of the paper.

[13] At this point, one might be tempted to offer this objection to the derivative-from-below proposal. On that proposal, body *a* does not qualify as having non-zero classical instantaneous velocity until it has moved a bit from the location it occupies at t=0 (so that its trajectory's time-derivative from below is non-zero). Until it has moved some distance, its classical instantaneous velocity (on the derivative-from-below proposal) must still be zero. But if it must change its position *before* it can be said to have non-zero velocity, then that change in position cannot be explained by its having non-zero velocity then; it must move while its velocity is still zero! If body *a* must move before it has non-zero velocity, but body *a* must have non-zero velocity before it can move, then body *a* can never be put in motion.

However, this objection is ill conceived. There is no first moment after t=0; at each moment after t=0, there are earlier moments after t=0. The derivative-from-below proposal entails that at each moment after t=0, body *a* has non-zero classical instantaneous velocity in virtue of its positions at earlier moments after t=0. For any time T at which body *a* occupies a position other than its pre-collision location, there are earlier moments at which it also does so, giving it a non-zero instantaneous velocity at T on the derivative-from-below proposal. There is a last moment at which it occupies its pre-collision location, and at that moment (namely, t=0), its velocity (on the derivative-from-below proposal) is zero. But at each later moment, it has non-zero velocity (on the derivative-from-below proposal) and it has moved from its pre-collision location. It does not have to move before the first moment when it has non-zero velocity because there is no first such moment.

14 At least not in the standard sense of integration. There is a mathematical device for representing such an acceleration—the "delta function," which is actually not a function. It is a functional. Whereas a function associates a number with a number, a functional associates a function with a number. The delta "function" can be integrated, but this fact is largely a matter of notational convention, since it involves using the integration sign to represent an operation that is not standard integration, but rather anti-differentiation "in the distributional sense." Although this procedure for manipulating the delta function is mathematically rigorous, it is difficult to see how it would allow instantaneous acceleration in a momentary collision to play anything like its traditional causal role. As Zemanian says, "One cannot assign instantaneous values to a distribution, and consequently the problem of physically interpreting such values does not arise" (1965, 2). This observation may comfort a mathematician, but it does not encourage us to use the delta function to understand instantaneous acceleration within a causal interpretation of classical physics.

15 This fact inspires one argument that classical mechanics should not be interpreted causally.

16 My objection to such an idealization may appear less querulous when it is borne in mind that the infinite forces required by momentary collisions have typically been viewed with extreme suspicion by physicists who interpret classical physics in traditional causal terms. (See, for instance, Thomson and Tait 1888, 1.)

17 There are continuous curves that are nowhere differentiable, such as the Blancmange function (see Spivak 1980, 144–45 and 474–75). Some philosophers might contend that intuitively, a body pursuing such a trajectory can possess an instantaneous velocity everywhere, even though Russell's reductive account dictates that the body has no instantaneous velocity anywhere. However, although such a trajectory is continuous, any finite interval along it contains infinitely many points like the ramp function's $t=0$. Hence, like the ramp function, the Blancmange function cannot be a body's trajectory in classical physics (interpreted causally).

18 Unless (as John Carroll pointed out to me) we are discussing a body's $v(t_0)$ where t_0 is the first moment of time (if classical physics permits time to have a first moment).

19 For discussion of analogous continuity principles regarding energy and momentum, and their importance in the development of electromagnetic field theory, see Lange 2002a.

20 Strictly speaking, the body consisting of the bullet and block existed—in two detached parts—even *before* the collision. Any system of bodies functions in classical physics as a single body, the trajectory of its center of mass governed by Newton's second law of motion. So strictly speaking, there are no newly created bodies in classical physics. The time-derivative at t_0, taken from below, of the bullet-block system's center of mass is the "new" body's velocity t_0.

21 Likewise, this principle entails that a body's $v(t_0)$ cannot be a cause of its change of position between t_0 and $(t_0 + \Delta t)$ unless $v(t_0)$ is either a cause of

$x(t_0)$ or a cause of $x(t_0 + \Delta t)$.

[22] Even if backwards causation is admissible in exotic cases, this is not an exotic case. Note also that E at a given location and the body's occupying that location $x(t_0)$ at t_0, along with the body's mass and charge (and perhaps the absence of other forces), form a *complete* cause of $a(t_0)$. Presumably, then, if $a(t_0)$ is a relation's holding, then *each* relatum must have some member of the complete cause as a cause. But it is difficult to see how this can be.

[23] Among those who have defended views roughly along these lines are Blackburn (1990), Broad (1933, 1:269–72), Foster (1982, 66–67), Langton (1998, 176), Lewis (unpublished), Mackie (1973, 150–52), Mumford (1998, 132–33), Poincaré (1902/1952, 161), Price (1953, 322), and Worrall (1989).

[24] For a critique of this view of electric charge, see Lange 2002a, 86–90.

[25] Of course, essentialists such as Ellis and Shoemaker disagree with the traditional view. For discussion, see my 2004.

[26] Tooley (1988, 238–39) defines "velocity" as whatever property stands in such relations. On Tooley's view, though, it is not essential to whatever property in fact stands in these relations that it does so. That property merely qualifies as velocity in virtue of standing in these nomological relations.

[27] For example, a body manifests no fragility by breaking apart when shaken around, if the shaking causes a massive quantity of TNT to explode. A good deal more could be said about this, but none of that is relevant to our concerns here.

[28] For example, a world operating according to classical physics (interpreted causally) contains no field that affects trajectory by automatically making any body located within it move at a uniform 2 cm/s, since such a field would sometimes have to produce an instantaneous, finite change in a body's velocity, and as we saw in connection with collisions, this cannot be accommodated within classical physics (interpreted causally).

[29] Perhaps the circularity can be avoided by defining the body's having uniform velocity in the interval $[t_0, t_0 + \Delta t]$ not as the body's having the same instantaneous velocity at each moment in this interval (which creates circularity if "uniform velocity" appears in the definition of "instantaneous velocity"), but rather as the quantity $[x(t_2) - x(t_1)] / [t_2 - t_1]$ being the same for every two distinct moments t_1 and t_2 in the interval. This may even be Maclaurin's strategy. (The two definitions of uniform velocity are equivalent in a possible world operating according to classical physics, interpreted causally, only because instantaneous velocity there is always differentiable, as we have seen.) But were the body moving uniformly (by this definition) in $[t_0, t_0 + \Delta t]$, would its $v(t_0)$ be what it actually was? I see nothing to guarantee it, and this spoils the proposed definition of $v(t_0)$, as I suggest in the next paragraph.

[30] This is so even if the world's laws give a body a very large chance of behaving classically and only a tiny chance of having its subsequent trajectory fixed randomly. Notice also that even in this possible world, the logical truths I gave in section 3 (that certain integral equations hold *if* the body has a well-defined trajectory, instantaneous velocity, and instantaneous acceleration) are true—because no body has a well-defined instantaneous velocity.

Consider instead an occasionalist world where God decides where each body will be at each instant. Suppose that God decides in advance that a given body will pursue a trajectory having a time-derivative from above at t_0. Then by my account, the body possesses a classical instantaneous velocity at t_0, although God's will rather than that velocity is a cause of its subsequent trajectory. Velocity's causal role in a world operating according to classical physics is beholden to contingent laws of nature that are not laws in an occasionalist world.

[31] I do not regard the right side of this biconditional as specifying what *makes* the left side true.

[32] My rationale for denying this subjunctive conditional is similar to the familiar rationale for denying that the principle of conditional excluded middle applies to counterfactuals. See Lewis 1986, 329–31.

[33] At t_0, it is true that were the body to continue to exist after t_0, its trajectory at t_0 *might* have a well-defined time-derivative from above. However, on my proposal, it does not follow that the body at t_0 might have a $v(t_0)$. Now here's an objection: if the body turns out in fact to move along a differentiable path after t_0, then *after* t_0, it seems true that were the body to continue to exist after t_0, its trajectory would have a well-defined time-derivative from above at t_0. How can the subjunctive conditional "Were the body to continue to exist after t_0, then its trajectory would have a well-defined time-derivative from above at t_0" be false at t_0 but true after t_0? I reply that the quoted words are ordinarily taken as standing for different propositions at and after t_0. At a given moment, the words "$p > q$" are ordinarily taken as demanding that p be added to some (context-sensitive) subset of the facts concerning the universe's history through the moment at which the words are uttered. So when the subjunctive conditional "Were the body to continue to exist after t_0, then it would have a well-defined time-derivative from above at t_0" is uttered after t_0, the fact that the body happens to have a differentiable trajectory after t_0 is typically part of what p is supposed to be added to, whereas this is not the case when these words are uttered at t_0.

When I say "$p > q$ is true *at time* t_0" (as I do in my proposed analysis of velocity), I do not mean to suggest that the same proposition changes its truth value over time. Rather, I am using "*at time* t_0" to indicate something about the context in which the subjunctive conditional should be entertained. (See Lycan 2001, 158–59 for a similar view.) Subjunctive conditionals are notoriously context-sensitive. So any analysis that employs them must (and is entitled to) specify the relevant context.

[34] In my 2000, 2002b, and 2005, I explain how the facts about which truths follow from the laws, and which truths are accidents, supervene on the facts about which subjunctive conditionals are true.

[35] For examples of such arguments, see Tooley 1977, Carroll 1994, and Lange 2000. Of course, some of the examples used in those arguments take for granted that velocity is a categorical property. But no matter.